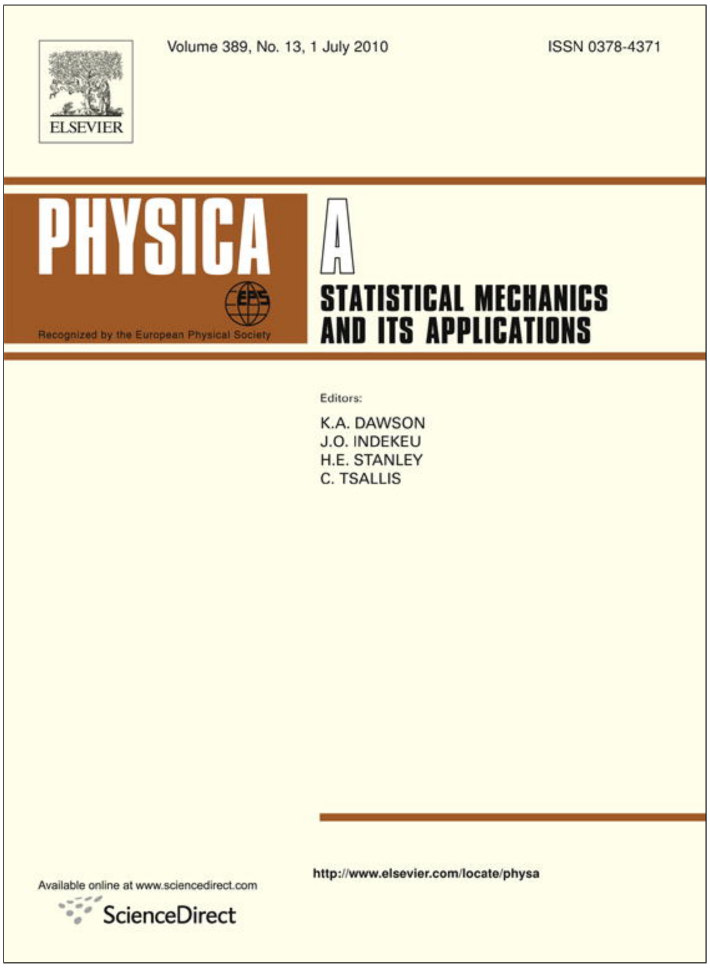
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On the energy content of a money unit

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ABSTRACT

In this paper, the concept of "productive energy" as a substitute for conventional labour is examined historically, theoretically and empirically. Over the course of the last centuries, productive energy has been substituted for human, muscle- and brain-based work, providing the wherewithal for the phenomenal growth in material wealth in Western societies. In this era of rising energy costs and increasing energy scarcity, future growth appears to be compromised. To better understand the consequences for society, estimates of the energy content of a dollar's worth of output are provided for the US and Russia.

1. Introduction

Modern money creates great difficulties for both the performance and analysis of economic systems. The output of a production system is measured in arbitrary and changing money units, which is a real headache. As a well-known financier Lietaer [1, p. 254] writes: 'The world has been living without an international standard of value for decades, a situation which should be considered as inefficient as operating without standard of length or weight.' Could any objective foundation for establishing the unit of money be found?

Once there was belief that economic value is equivalent to labour, so that a unit of money is equivalent to unit of labour. This is known as *labour theory of value*, developed in classical political economy. However, there were problems with this view as discrepancies began to emerge. For example, the growth rate of the use of labour in production appeared to be less than the growth rate of output in developed economies, the fact which prompted scholars to look elsewhere.

The past two centuries witnessed the gradual replacement of human-based muscular work by inanimate forms of work as a source of motive power in material processes. Where workers once toiled, today, inanimate energy-powered machines now do. In the process, labour has been transformed into a supervisory input, overseeing the workings of these highly complex, and highly productive machines. Today, we are witnessing another fundamental transformation as control devices and information technology are rapidly transforming the workplace, rendering human forms of supervision redundant. Like the supervisors they replace, control devices and information technology are energy-based. Human neural circuits are energy-driven as are the processors that comprise the control devices.

The results are there for everyone to see. While the form it has assumed has changed over time, the fact remains that energy powered, powers and will continue to power work processes, and moreover, it powered, powers and will continue to power supervisory activity. These developments stand as testimonies to the universal character of energy as the basis of

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material processes and hence of the creation of value (see, for example, Refs. [2–9]). Over the course of the past centuries, productive energy has metamorphisized from its muscular form (over millions of years) to hydrocarbons, to steam and more recently to electricity. This raises an important empirical question, namely, what is the energy content of a money unit? How much energy one needs to create every dollar of output? Has it increased over time, or decreased? Is value more or less energy dependant?

The development of thermodynamics in the 19th century, and its application to a number of fields, including biology, and psychology, contributed to a new intellectual tradition, one that put energy at the core of its analysis. Further developments in the 20th century, specifically in the field of chemistry, reinforced the universal character of energy as the basis of matter and indeed of all material processes. Political economy, the science of wealth, was no exception. Perhaps the most notable attempts at integrating energy in a meaningful way into the science of wealth were provided by Nobel Prize laureate Frederick Soddy [2] and Howard Scott [3]. Both viewed energy as "the" universal factor input. Later, others, in the tradition of Soddy [2] and Scott [3], argued that energy must be considered as the only source and measure of value [10,11], and moreover the concept of value itself should be based on energy, which is known as *energy theory of value*.

While labour and energy theories of value have a long history, the dominant theory today (neoclassical) maintains that labour and capital, which is a money estimate of all production utilities, are assumed to be *production factors* — factors that are the universal sources of value. Energy as itself is not considered in the theories of production; energy carriers are being regarded as conventional *intermediate products* that contribute to the value of produced commodities by adding its cost to the price. Why then is energy not a source of value? This controversy stands and has been discussed for decades. Could any production factor relating to consumed energy be found?

In this paper, encouraged by interest in the applications of physical methods to non-traditional problems [12], we attempt to provide a framework for analyzing these questions and issues. The phenomenon of production is considered from a physicist's point of view, taking the economists' concerns and preoccupations into account. For starters, it would be helpful to consider more thoroughly the role of energy in economic processes and isolate that part of overall energy consumption that contributes to adding value (as opposed to simply being value). In the next section, we examine the universal role of energy from an historical point of view, paying particular attention to the role of energy in production theory. It will be shown, that by virtue of the substitution effect, the true substitutive work of production equipment P has to be considered as a production factor in line with the conventional production factor of neoclassical economics: labour L. Section 3 discusses the main principles of the quantitative theory of production [13,14], which is based on the discussed law of substitution and includes both the new production factor P and the production factors of the conventional neoclassical economics: capital stock K and labour expenses L. The results in Section 4 provide estimates of 'the energy content of a money unit' for the United States and Russia.

2. The law of factor substitution

The role of a production system is to transform matter into 'useful' forms (dwellings, food, clothes, buildings, machines, transport means, sanitation, home appliances, machinery and other commodities) that support human existence. These items can be described in any number of ways, however it is important that whatever the description chosen, there is an unique and unifying measure of all services and commodities: value can be ascribed to every product, so that one can speak about production of things as well as about production of value. We use a conventional symbol Y to denote production of value for a nation, gross domestic product (GDP), which is reflected in national statistics. It is a measure of the current achievements of the economy as a whole — a measure of a multitude of fluxes of products, produced during a year.

2.1. Capital and the neoclassical concept of factor substitution

While considering the performance of the production system, early political economists introduced the concept of capital, which they defined as fixed material assets, that is infrastructure, industry buildings and basic production equipment — all that is used to produce goods and services. In US data, capital includes estimates of private fixed assets, government fixed assets and consumer durable goods. They believed in the productive force of capital and considered it as a source of value in line with labour. Consequently, output was defined as an increasing function of labour and capital, of which the Cobb–Douglas functional form is the best known [15]

$$Y = Y_0 \frac{L}{L_0} \left(\frac{L_0}{L} \frac{K}{K_0} \right)^{\alpha'}. \tag{1}$$

The scalar α' is a parameter of the production system. Eq. (1) formalizes that, in production of value, capital could act as a substitute for labour. Although the neoclassical concept of factor substitution was heavily criticised [16,17], it survives today as one of the foundations of the neoclassical theory of production. Minor issues, however, remain, such as the question of the proper empirical definition of capital and labour [18–22]. Other empirical issues include the so-called Solow Residual and the concept of total factor productivity, prompting some to consider such things as technology, human capital, stock of knowledge to name a few tested [23,24]. Others have argued in favour of including energy (or exergy) as a primary factor input [5,7,25].

2.2. The role of capital-machinery and equipment

To transform matter, work is required.[1] According to most models, work can be accomplished either by workers and/or by some external energy source (water, wind, coal, oil, et cetera). For example, to grind corn into flour, one can use either a hand mill, a water mill, a wind mill, or a steam mill. In the latter cases, the work performed by labour is replaced by the work of falling water, or wind, or heat.

Perhaps the first to describe the functional role of machinery in production was Galileo Galilei, who realized that all machines transmitted and applied force as special cases of the lever and fulcrum principle. According to science and technology historian Donald Cardwell [26], Galileo recognized that "the function of a machine is to deploy and use the powers that nature makes available in the best possible way for man's purposes … the criterion is the amount of work done – however that is evaluated – and not a subjective assessment of the effort put into accomplishing it" (pp. 38–39). The advantage of machines derives from their ability to harness cheap sources of energy since "the fall of a river costs little or nothing".

The relevance of machinery to economic performance was clearly recognized by Marx [27], who described the functional role of machinery in production processes in Chapter XV *Machinery and Modern Industry* of Das Kapital as follows:

> On a closer examination of the working machine proper, we find in it, as a general rule, though often, no doubt, under very altered forms, the apparatus and tools used by the handicraftsmen or manufacturing workman: with this difference that instead of being human implements, they are the implements of a mechanism, or mechanical implements (pp. 181–182). The machine proper is therefore a mechanism that, after being set in motion performs with its tools the same operations that were formerly done by the workman with similar tools. Whether the motive power is derived from man or from some other machine, makes no difference in this respect (p. 182). The implements of labour, in the form of machinery, necessitate the substitution of natural forces for human force, and the conscious application of science instead of rule of thumb (p. 188). After making allowance, both in the case of the machine and of the tool, for their average daily cost, that is, for the value they transmit to the product by their average daily wear and tear, and for their consumption of auxiliary substances such as oil, coal and so on, they each do their work gratuitously, just like the forces furnished by nature without the help of man (p. 189).

Hence, physicists and political economists both recognize the important role of machinery in production processes as having to do with the *substitution of labourer's work by the work of machines moved by external sources of energy*, while the extent of this substitution depends on the technology *per se*. It is important to keep in mind that while capital is a necessary factor input, work can only be replaced by work, or put differently, work cannot be replaced by capital. Note that by contrast with Smith and Marx who focused on physical labour, we shall consider all possible energy-driven activities of conventionally-defined workers including supervision of any kind, that is we use an extended concept of labour (human capital).

2.3. Estimating the effect of substitution on production of value

Judging by its role in both the history of economic thought and its place in modern production theory, labour stands as the key factor of production. However, what is also clear is that something else is also "working", something Marx referred to as "the substitution of natural forces for human force". Indeed, one could as far as to argue that after having understood this phenomenon, Marx could/should have examined how this "substitution" affects the production of value. For example, in an effort to better understand how this affects the creation of value, he could/should have compared two similar firms with different technology: one using L units of labour and P units of substitutive work, while the other using $L - \Delta L$ and $P + \Delta P$, respectively. If the products of both firms are considered to be identical, then the exchange values of the products on the market are equal, despite the difference in labour consumption, which provides the following identity:

$$-\beta\,\Delta L + \gamma\,\Delta P = 0,$$

where β and γ are the corresponding productivities. Here, β/γ corresponds to the work (by external sources) required to substitute a unit of human work holding production value constant.

In the general case, the work performed by labour L and productive energy P can be expressed in terms of the exchange value Y, allowing us to write the following equation

$$\mathrm{d}Y = \beta\,\mathrm{d}L + \gamma\,\mathrm{d}P. \tag{2}$$

The coefficients $\beta > 0$ and $\gamma > 0$ correspond to the value produced by the addition of unit of labour input at constant external energy consumption and by the addition of unit of work of production equipment at constant labour input, respectively. These are known as marginal productivities. The two production factors, work of labour and work of external sources of energy (inanimate), are substitutes giving new meaning to Adam Smith's view to the effect that labour is "the

[1] One should understand work as a process of energy conversion (i.e. from one form to another, for example, from the mechanical to the thermal form).

only universal, as well as the only accurate measure of value, or the only standard by which we can compare the values of different commodities at all times, and at all places''.

Thus, with account of substitution effect, *the generalised labour theory of value*, which also can be called *the generalised energy theory of value*, can be formulated.

3. A quantitative theory of production

This finding/formalization allows us to develop a new theory of production, one that is consistent with previous work [14]. The key contribution is the notion of substitutive work (productive energy) as a production factor and the key development is the effect of substitutive work (productive energy) on production of value as well as a more consistent description of actual economic growth. In this section, we discuss the main principles of the phenomenological theory of production [13,14] based on the principle of substitution, include both the new production factor P and the production factors of conventional neoclassical economics: capital stock K and consumption of labour L, and demonstrate the possibility of a consistent description of economic growth.

3.1. The concept of substitutive work

It has been suggested [28,29] that the total consumption of *primary energy* carriers (for simplicity one speaks about consumption of primary energy[2]) E can be separated into two parts according to its role in production processes. One part of the energy carriers is used to perform the ''real work'' of production equipment. We will refer to this part as *primary productive energy* E_P which is directed to substitute human work. The second part of energy carriers is used by households and firms to get light and heat. Following Ayres et al. [29], we will refer to the second as quasi-work.

In economic terms, all consumed energy carriers are intermediate products that contribute to the value of the made products, as any other intermediate product participating in the production by adding the cost of the carriers to the price. However, the part of the consumed energy, more exactly the part of primary productive energy E_P, after many conversions and losses accounted for as P is used by the production equipment to complete the same operations as labourers,[3] ought to be recognized also as a production factor. Productive energy is a service provided by the production equipment—a capital service like the labour service. The quantity P is a small part of the primary productive energy E_P, and the coefficient of efficiency P/E_P depends on the applied technology and apparently is different in different situations.

By way of an introduction, it can be argued that substitutive work is, in essence, the work of capital equipment. It is the part of energy consumption that is used in lieu of labour [14]. Combined with the work of labour, we get the total work during the production in energy units

$$A = P + hL, \tag{3}$$

where h is the work (in physical sense, in energy units) done by unit of measurement of efforts of workers.

The true measure of labour is the work (in the physical sense, in energy units) done by a worker. However, because labour L is measured by working time (in hours per year, for example), it is important to estimate the work h done by a worker per hour. According to Rivers and Payne [30], in a sedentary state, the human organism (an adult male) requires about 2500 kcal/day or about 10^6 kcal/year $\approx 4 \times 10^9$ J/year. Extra activity would require additional energy—energy expended by a working man can be up to two times as much as the energy of a resting man. As such, one can estimate the upper level of the work done by a worker as 100 kcal/h or 4.18×10^5 J/h. The estimates can differ according to time and place. The possibilities of the ''the human engine'', for example, were lower in the earlier times, as was shown by Fogel and Costa [31] on the base of historical data for France and Britain for years 1785 and 1790.

The ratio of substitutive work of production equipment to the energy estimates of human efforts shown in Fig. 1 can be seen as a measure of technological progress. To use external energy in the production, one would need to have available sources of energy and appliances that use energy.

As devices have to be invented, produced and installed in order that energy can ''work'', it stands to reason that the supply of productive energy is determined by science in general, by research, and by human imagination (i.e. on how to use energy

[2] While we use the term ''energy consumption'', precision would dictate that the word *consumption* should be replaced by the word *conversion*. Energy cannot be *spent/consumed* in manufacturing processes, but rather is transformed into other forms: chemical energy in thermal energy, thermal energy in mechanical energy, mechanical energy in energy thermal and so on. For an estimation of quantity of possible transformation of energy (work), the concept of exergy is used.

[3] To avoid misunderstanding, note that a labourer in this context is a person who spends the force of his muscles or brain to do any of a number of tasks. To illustrate, here, according to Ref. [28], is a list of work tasks that have been replaced and continue to be replaced by industrial equipment:

1. *Efforts on displacements of substances and bodies (including humans own bodies)* were substituted by the work of animals, wind and moving steamer engines in the past. Now they are substituted mainly by the work of self-moving machines, automobiles, trucks, aeroplanes and other mobile equipment.
2. *Efforts on transformation and separation of substances and bodies* are efforts in the production of clothes, tools, different appliances and so on—much, if not all, manufacturing. The animal drive, wind drive, water drive and steam engine drive were used to do work instead of humans in previous centuries. To the middle of the twentieth century, the same work is being mainly done by machine with electric drive [29].
3. *Efforts on sense-based supervision and co-ordination, development of principles of organisation* were considered as essentially human functions up to recent times. Now the work of the brain is being substituted by information processors driven by electricity.

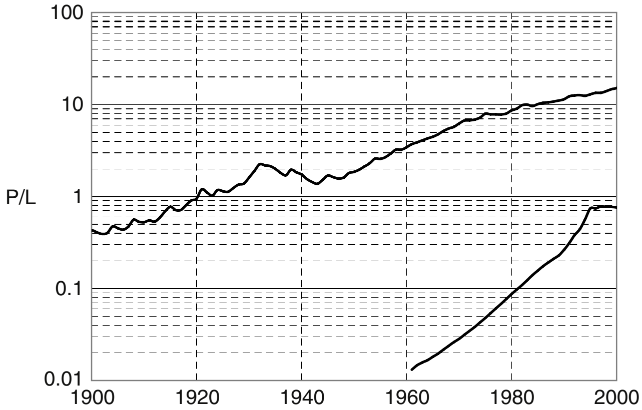

Fig. 1. The ratio of substitutive work to the estimates of human effort. The ratio for the US economy (the upper curve) was taken from [14] while the ratio for the Russian economy (the lower shorter curve) was estimated using recently-collected data (http://ecodynamics.narod.ru/Russia/table.doc).

for production). There can be no doubt that historically speaking, the development of new machine technologies has, via the substitution effect, vastly increased labour productivity. Put differently, energy-related inventions and innovations have resulted in a marked increase in energy consumption.

It is our view that substitutive work or *productive energy* P should be seen as source of value on a par with the production factors of conventional neoclassical economics — capital K and labour L. The universal importance of labour and capital for economic performance is widely recognized. The important role of energy in production is, in our view, indisputable. However, while there are data on industrial energy consumption, there are none on substitutive work (genuine productive energy), which drives production equipment. In order to surmount this problem, a new method of estimation was developed [28,29].

3.2. The three-factor production function

According to classical and neoclassical traditions, production of value Y can be formalized in terms of production factors which are as such considered as sources of value. In the neoclassical approach, these consist of labour L and capital K. In keeping with Sections 2.2 and 2.3, properly-defined substitutive work should be regarded as a production factor as well, while the total consumption of energy carriers E ought to be considered as an intermediate product in complete agreement with economists' view. According to Eq. (2), output Y is an increasing function of labour L and work of production equipment P. Also, we must also take account of production equipment measured by its value K (capital stock), making for a situation in which the production function can be considered as a function of three arguments.[4] As pointed out in Section 2, we consider that labour L and work of production equipment P are substitutes, but are complements to production equipment, making it necessary to separate the function into two parts.

$$Y = \begin{cases} Y(K) \\ Y(L, P), \end{cases} \qquad \mathrm{d}Y - \Delta \mathrm{d}t = \begin{cases} \xi(K)\,\mathrm{d}K \\ \beta(L, P)\,\mathrm{d}L + \gamma(L, P)\,\mathrm{d}P \end{cases} \tag{4}$$

where $\Delta \mathrm{d}t$ is the change in output brought about by a change in the characteristics of the production system (the technological and structural changes). The quantities ξ, β and γ are the marginal productivities of the corresponding production factors.[5] Ceteris paribus, the marginal productivity ξ corresponds to the value produced by adding a unit of capital; β and γ correspond to the value produced by adding a unit of labour and by adding a unit of energy, respectively.

One can specify (4) by requiring it to be universal, that is independent from the initial point (the principle of universality) and by assuming homogeneity.[6] Thus, in its simplest form, when the production process is viewed as a collection of equipment (measured by its value K), getting its ability to act from labour (L) and capital services (P) inputs, our production

4 Three-factor models were considered by others [5,25]. The success of the earlier attempts was limited by the choice of total energy consumption as an argument of production function. For example, Kümmel [5] defines the production function as a function of three variables $Y = Y(L, K, E)$, whereas, in contrast to the discussed theory, considering the total consumption of primary energy E as an argument of production function. This choice of production factors allows the substitution of different factors by each other and does not allow one to determine the roles of the production factors K, L, and E in the theory. The specific representation of the production function by Kümmel [5] is not acceptable from the general point of view: it does not satisfy the principle of universality: parameters depend on the starting point—while allowable, it biases the results.

5 It is important to carefully define the notion of marginal productivity so as not to confuse it with those derived from different definitions of the production function.

6 The discussion of the effects of universality and homogeneity on the form of production function can be found in Section 6.3 of Ref. [32].

function can be specified as:

$$Y = \begin{cases} \xi K, & \xi > 0 \\ Y_0 \dfrac{L}{L_0} \left(\dfrac{L_0}{L} \dfrac{P}{P_0} \right)^{\alpha}, & 0 < \alpha < 1 \end{cases} \tag{5}$$

where L_0 and P_0 correspond to labour and capital services in the base year. This approach offers two complementary descriptions of the production of value. The first relates output to the amount of production equipment (capital stock) while the second describes the process of production through the property of the same equipment to attract labour and energy (labour and capital services). The first (5) is analogous to the production technology used in the Harrod–Domar growth model [33–36], while the function in the second resembles the Cobb–Douglas production function (1). The productivity of the capital stock ξ and the index α in Eq. (5) are parameters of the production system itself and related to each other. It needs reminding that in the conventional neoclassical approach, capital plays two distinctive roles: capital stock as the value of production equipment and capital service as a substitute for labour. These roles are ascribed to different variables in our theory: Eq. (5) contains productive energy P as a capital service and capital stock K as a measure of amount of production equipment.

This yields the following expressions for the relevant marginal productivities:

$$\xi = \frac{Y}{K}, \quad \beta = Y_0 \frac{1-\alpha}{L_0} \left(\frac{L_0}{L} \frac{P}{P_0} \right)^{\alpha}, \quad \gamma = Y_0 \frac{\alpha}{P_0} \left(\frac{L_0}{L} \frac{P}{P_0} \right)^{\alpha - 1}. \tag{6}$$

The productivity of the capital stock ξ can be reduced to the index α, which is a parameter of the production system and related to the characteristics of technology. For this reason it will be referred to as the technological index. The growth rate of the productivity of capital stock ξ is, as such, related to changes in the technological index ([32], Chapter 6, Equation 6.17)

$$\frac{1}{\xi} \frac{d\xi}{dt} = \ln \left(\frac{L_0}{L} \frac{P}{P_0} \right) \frac{d\alpha}{dt}. \tag{7}$$

In the multi-sector approach (input–output model), changes in the technological index are related to aggregate sectoral technological change and with the difference in the growth rates across sectors, as can be seen from Equation 8.21 in Chapter 8 of Ref. [32]. The technological index itself can be estimated using all available information about the technological performance of the production system. Moreover, a condition regarding the optimal use of production factors enables us to establish a relation between the parameter α on one hand and the shared costs of production factors on the other [14]. This provides an alternate means of estimating of the technological index.

3.3. *Application to the US economy*

3.3.1. *An empirically-consistent description of growth*

To illustrate the theory, we use time series for output Y, capital K and labour L for the US economy in years 1890–2000. The relevant time series are available from US government web sites and are presented in the Appendix of Ref. [14]. The empirical relationships are shown in Fig. 2 in line with total primary consumption of energy E, which is the amount (in energy units) of energy carriers, including primary productive consumption of energy. Our theory has two other variables: the productive energy P and technological index α. Fortunately, methods for estimating of capital services—the third production factor P and the index α for given time series of Y, K and L can be developed, so that one can find [14] such values of both capital services P and the technological index α, shown also in Fig. 2, that calculated values of output coincide with the empirical ones. The index α corresponds to the share of expenses needed for using capital services in the total expenses for production factors and can be evaluated using estimates of the cost of consumption of production factors. The different estimates of the technological index α were found to be consistent [14]. Thus, the model has no arbitrary parameters.

Given that our production function (5) provides a consistent description of the past empirical situation for years 1900–2000, it can be extremely useful as a forecasting tool. Lastly, one has to forecast the changes in the production system itself, denoted by changes in α owing to technological and structural modifications and the future values of production factors. The technological index α changes slowly and can be considered constant during decades; as one can see in Fig. 2, the major changes of the technological index are trigged by extraordinary events similar to the Second World War in years 1940–45.

3.3.2. *"Stylised" economic growth*

Our model can be applied to a simple case, where inputs grow exponentially.

$$K = K_0 e^{\delta t}, \qquad L = L_0 e^{\nu t}, \qquad P = P_0 e^{\eta t}. \tag{8}$$

Substituting these into (5), output can be written as:

$$Y = Y_0 e^{[\nu + \alpha(\eta - \nu)]t} = Y_0 e^{\delta t}. \tag{9}$$

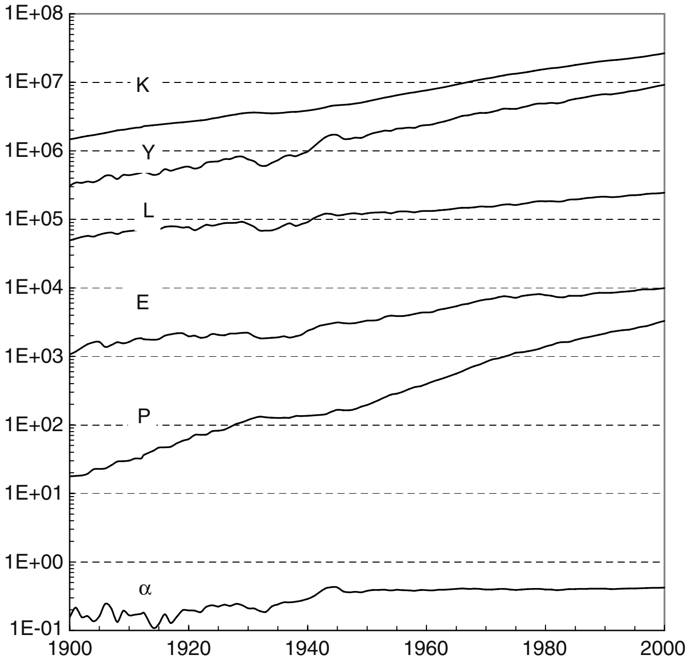

Fig. 2. An empirically-consistent description of US economy growth. Fig. 2 shows the empirical estimates of production of value (GDP) Y, millions of 1996 dollars per year; production equipment (capital stock) K, millions of 1996 dollars; consumption of labour L, millions of working hours per year. The values of substitutive work (productive energy) P, 10^{16} joules per year, and the technological index α were calculated [14] to correspond to these values of Y, K and L. Total consumption of primary energy carriers E, 10^{16} joules per year, is also shown.

In this version, the growth rate of output is equal to the growth rate of capital stock, as, indeed, one can see in Fig. 2 for the "calm" period of years 1950–2000, and is related to the growth rates of labour and capital services. The resulting expression implies that the growth rate of labour productivity is determined by the difference between the growth rates of energy and labour $\alpha(\eta - \nu)$. In the period 1950–2000 for the US economy (see Fig. 2), $\delta = 0.0316, \quad \nu = 0.0146, \quad \eta = 0.0588$. The average growth rate of output 0.0329 is approximately equal to the growth rate of capital $\delta = 0.0314$. One can estimate the contribution of labour and capital services to the growth of output. Given that α assumes a value of 0.4, their contributions are $(1 - \alpha)\nu \approx 0.0088$ (labour growth) and $\alpha\eta \approx 0.0235$ (capital services growth).

As capital stock is the means of channeling production factors, an increase in the consumption of the production factors will be correlated with an increase in capital stock. Further, one can also separate the growth rate of capital stock δ in the growth rate of capital services η to get the breakdown of the growth rate of output in conventional terms: the contribution from the labour growth is $(1-\alpha)\nu \approx 0.0088$, the contribution from the capital growth is $\alpha\delta \approx 0.0126$, and the contribution from the total factor productivity is $\alpha(\eta - \delta) \approx 0.0109$. Traditionally, the latter refers to changes in the production system itself, but, according to our interpretation, they represent contributions of the growth of production factors in total factor productivity. There is no contribution from changes in the production system itself, when exponential growth (7) and (8) is assumed.

3.4. What is the productivity of capital?

Our analysis also allows us to examine the marginal productivities of both labour and productive energy, β and γ, shown in (2) and (4). The explicit forms of the marginal productivities are given by (6) and are estimable. Note that these quantities, by virtue of Eqs. (5) and (6), are related:

$$\xi = \beta \frac{L}{K} + \gamma \frac{P}{K}. \tag{10}$$

The estimates of ξ, $\beta \frac{L}{K}$ and $\gamma \frac{P}{K}$ for the US economy [14] are shown in Fig. 3. For 1950–2000, the average capital-stock marginal productivity was (0.309 ± 0.035) year^{-1}, whereas the average of the right hand side of Eq. (9) is (0.320 ± 0.041) year^{-1}. The values of the marginal productivity is remarkably close to the averaged bulk productivity Y/K, which is (0.318 ± 0.010) year^{-1}. This is evidence that the capital marginal productivity does not depend on K.

Thus, the marginal productivity of capital can be expressed as the "sum" of the marginal productivities of labour and capital services and stands as a fundamental characteristic of the production system. Production equipment (capital stock) is what attracts labour and capital services, defining the process. The productivity of capital is, in fact, the productivity of labour and energy.

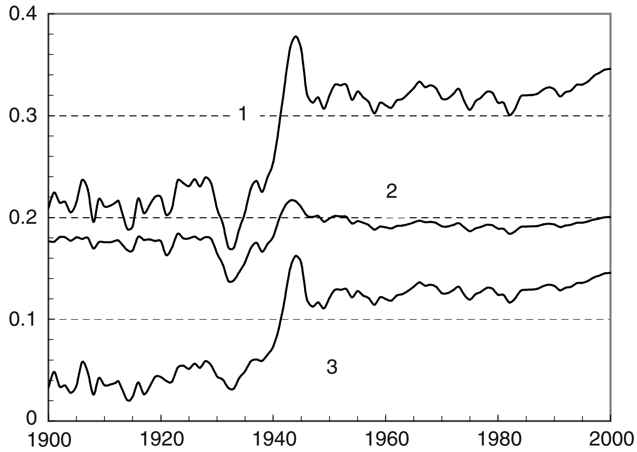

Fig. 3. Marginal productivities—US The solid lines 1–3 are the estimates of: ξ, $\beta L/K$ and $\gamma P/K$, correspondingly, in year^{-1}.

4. Farewell to the dream?

Taking our analysis further, one could ask whether there is sole related-to-energy quantity, which like labour in Marx's Labour theory of value appears to be a source of all value (output). Such a quantity would include the true work of production equipment or productive energy. While productive energy is a small part of the total amount of consumed energy, it is the only part directly concerned with production of value. The total consumption of energy or exergy could also be considered, as was done by others [37–39], however, in this case, we would lose universality: the production of value would depend on the efficiency of conversion of energy in production processes.

One would expect that total amount of work, including properly accounted labour work and work of production equipment, could be an absolute measure of value. The total work on the production of value is the sum of productive energy and the work of workers, so that the properly accounted work per unit of time can be written as

$$A = P + hL. \tag{11}$$

This work produces "useful" changes in our environment (in the form of useful commodities and services) that can be estimated by the production of value[7] for unit of time Y (in money units, for year, for example)

$$Y = \beta L + \gamma P = \gamma \left(P + \frac{\beta}{\gamma} L \right). \tag{12}$$

From here, it is simple to derive the genuine work required to produce a good or service expressed in terms of monetary value or, in other words, the "energy content" of a money unit

$$\frac{A}{Y} = \frac{1}{\gamma} \frac{P + hL}{P + (\beta/\gamma) L} = \frac{1}{Y} (P + hL). \tag{13}$$

Expressed in these terms, estimates of the "energy content" of a money unit for the United States and Russian Federation economies are shown in Fig. 4.

The "energy content" of the dollar is (1–2) $\times 10^5$ joules per dollar of 1996; its mean value in the last years of the century (1960–2000) is 1.4×10^5 J.[8] 'The energy content' of the 2000 Rouble is less: the mean value for the same years (1960–2000) is 0.1×10^5 J. "The energy content" of the Dollar is 14 times that of the Rouble, whereas the exchange rate was about 30 Roubles for Dollar. While only speculative, we believe that these differences can be attributed to different values for h in the

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7 Though the concept of value, as a specific concept in economics, does not need to be reduced to a physical concept, one can nonetheless find analogies in thermodynamics. The environment can be considered to be a thermodynamic system, and by performing work (creating value), the economy reduces the entropy of the natural environment, so that value can be related to entropy with the reverse sign. The properly organised work of the production system is needed to transform the natural environment into an "artificial" one. The usefulness of application to thermodynamics for a deeper understanding of the process of production in economic terms was recognized by many researchers [7,11,38,39]. Beaudreau [7], for example, considers the work of production equipment W and another factor, called organization, which is considered as something different from work. If one excludes the discussion of process of conversion of consumed energy carriers into work and the second-law efficiency for simplicity (this is mainly a technical problem, which is not universal), work W is identical to productive energy or genuine work of production equipment, which is discussed in this paper, and the organization, which can be corresponded to labour L as supervision, is also apparently real work, which requires the input of energy. The two factors, correspondingly, are called inanimate and animate work by Beaudreau [7]. It seems that output now can be considered as a function of two factors, $Y = Y(W, L)$, but Beaudreau [7] identifies output and primary work of production equipment, including the second-law efficiency into the discussion. However, output measured as added market value is apparently different from the primary work; it ought to be defined independently.

8 Note that these estimates are naturally lower than "the total exergy or emergy content" [37,38] when all "previous" expenditure of energy needed for production "from the very beginning" are accounted. Such estimates includes all losses of energy during production.

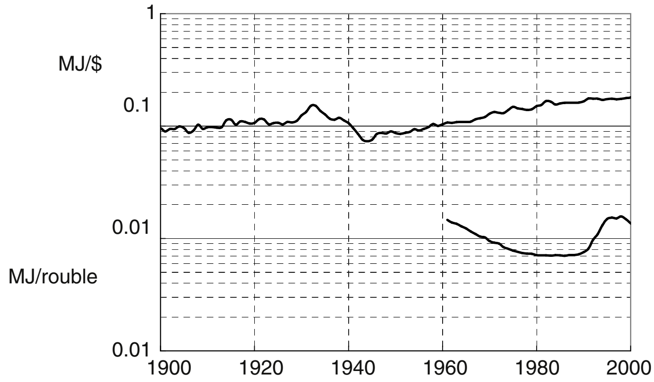

Fig. 4. The "energy content" of money units. The upper and lower curves show the "energy content" of the US dollar (1996) and Russian rouble (2000), correspondingly.

American and Russian economies. One could also attribute this to the absence of purchasing power parity (disequilibrium exchange rate). Lastly, it should be noted that the Russian data is, in general, less reliable than US data.

The dream was that the "energy content", could be an absolute measure of value, and, at the first glance, the values of the "energy content", shown in Fig. 4, appeared to be constant in time. However, it is not so: when the contribution of the substitutive work is dominating, which was the case in the US in the second half of the last century (see Fig. 1), the expression (13) for "the energy content" can be written as

$$\frac{A}{Y} \approx \frac{P}{Y}, \quad P \gg hL.$$

So as substitutive work increases faster that output, this shows, that "the energy content" is an increasing function of time. At the exponential growth, when the variables are given by Eqs. (8) and (9), "the energy content" of a money unit grows as

$$\frac{A}{Y} \sim \mathrm{e}^{(\eta-\delta)t}. \tag{14}$$

For the US in the second half of the last century, for example, $\eta - \delta = 0.0272$. Indeed, one can see the increase of "the energy content" on the plot of Fig. 4 for the USA economy after year 1950.

It could be the case that the increase of "the energy content" of a money unit, also as the pulsation of this quantity (see Fig. 4) was related to the change in labour's contribution (in energy units) contrary to our assumption. A more detailed account of labour costs (in energy units) could help resolve the problem. As this makes clear, more work (theoretical and empirical) is necessary.

5. Conclusion

In this paper, we have argued that "the energy content" of a money unit cannot be estimated without a clear understanding of the proper role of energy in economic performance. We argued that there is such a quantity, which has a meaning of pure substitutive work and can be defined as a production factor, that provides a consistent description of empirical situations. Moreover, the approach presented here provides a framework in which the contrasting points of view on the role of energy in production of value are reconciled. Indeed, energy carriers remain the intermediate products economists refer to, but part of the consumed energy plays a crucial role in the production of value. This part of energy is a substitute for the work of workers, and the production of value, indeed, can be reduced to the expenditure of energy, as the biophysicists [11,12] have argued. This extension of the labour theory of value is closely related to conventional neoclassical theory where capital and labour are the main sources of production of value. Introducing a third production factor as substitutive work or productive energy, our theory specifies the way to view the role of energy in the production of value.

The results allow us to present a method of calculating "the energy content" of a money unit. In contrast to the earlier approaches [37,38], our way of estimating excludes from the accounting all possible losses and delivers the pure work of workers and production equipment. Estimated as such, exergy or energy "content" of a money unit does not depends on the characteristics of production processes, in particular, on the efficiency of production processes, which is not well known and varies in different situations, and serves to impede and confuse any possible comparison. Despite difficulties and uncertainties involved in energy accounting, the amount of pure work needed to produce a unit of value, was calculated for two major economies: the United States and Russian Federation. The energy "contents" of the dollar and the rouble, taking into account the exchange rate, were found to be remarkably similar confirming the chosen methodology and the theory of production itself. However, there remains the question: why "the energy content" of a money unit grows in time?

One can think that understanding the proper role of energy in economic processes would help to analyse economic processes. It seems the new ideas are penetrating into a wider social mind. As columnist Martin Wolf put it[9] in 'Financial Times': "Neoclassical economics analysed economic growth in terms of capital, labour and technical progress. But, I now think, it is more enlightening to view the fundamental drivers as energy and ideas. Institutions and incentives provide the framework within which the development and application of useful knowledge transforms the fossilised sunlight on which we depend into the stream of goods and services we enjoy". We think that understanding the proper role of energy in economic processes opens a way to a physical interpretation of social phenomena.

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