Physica A 463 (2016) 111–122

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

An elementary model of money circulation

Vladimir N. Pokrovskii [a,*], Christophe Schinckus [b]

[a] Moscow State University of Economics, Statistics and Informatics, Moscow, 119501, Russia
[b] University of Leicester, UK

HIGHLIGHTS

- A set of equation for money circulation in the production system is formulated.
- The contribution to Gross Domestic Product from the bank system is defined.
- The relation of 'the quantity theory of money' is justified.

ARTICLE INFO

Article history:
Received 10 June 2015
Received in revised form 10 January 2016
Available online 19 July 2016

Keywords:
Bank system
Efficiency of bank system
Money circulation
Quantitive Quantity theory of money
Bank credit
Endogenous money

ABSTRACT

This paper investigates money circulation for a system, consisting of a production system, the government, a central bank, commercial banks and many customers of the commercial banks. A set of equations for the system is written; the theory determines the main features of interaction between production and money circulation. Investigation of the equations in a steady-state situation reveals some relationship among output of the production system and monetary variables. The relation of quantity theory of money is confirmed, whereas a new concept of the efficiency of the system is introduced.

© 2016 Published by Elsevier B.V.

1. Introduction

There is no doubt that the real and the financial sides of the economy are closely connected with each other, which give a base to design a monetary macroeconomic models that incorporate banks, debt and money in such a way, as, for example, described in many details by Godley and Lavoie [1]. The literature dedicated to the link between production and money circulation is huge and includes a variety of approaches [2,3]. It is out of our scope to review and discuss numerous books and papers devoted to the problem; a tremendous list of literature can be found in the already cited monograph [1]. An extra view on the money problem gives econophysics, but it restricts itself mainly to discussion of microeconomic problems of distribution wealth and money, as can be read in reviews [4,5]. We are going to follow the macroeconomic line of Godley and Lavoie, aiming to formulate and investigate a system of equations for the simplest system, consisting of the government and many customers of commercial banks—producers and consumers, but trying to approach to the problem from the other side, beginning with presentation of a simple three sector production subsystem running in the money environment, created by the central and commercial banks (see Fig. 1).

The core of the system is a production subsystem, which creates real wealth of the society and is considered as an originator of value. It is assumed, that the production subsystem consists of three sectors: the first sector creates basic

* Corresponding author.
E-mail address: vpok@comtv.ru (V.N. Pokrovskii).

http://dx.doi.org/10.1016/j.physa.2016.07.006
0378-4371/© 2016 Published by Elsevier B.V.

production equipment (K—production capital), the second one creates non-material intermediate products (S), consumed by the other two sectors and stored in warehouses and depositories for the future production and non-production consumption, and the third sector creates products for direct consumption by humans (C) in accordance with earlier description [6]. Due to the input–output theory of Leontiev [7], one knows that the output of each sector is needed to maintain production of, generally speaking, all other sectors, so that the gross outputs X_K, X_S or X_C, are generally distributed among three sectors, and the balance relation for the products can be written as

$$\begin{aligned} X_K &= X_{KK} + X_{KS} + X_{KC} + I, \\ X_S &= X_{SK} + X_{SS} + X_{SC} + G, \\ X_C &= C, \end{aligned} \tag{1}$$

where I, G and C are components of final output, planned for sale beyond the intermediate production usage; these quantities are estimation of value (in money units) of created commodities: the quantity $I = I_K + I_S + I_C$ is estimation of the value of the investment products, distributed over the three sectors; the quantity G is estimation (in money units) of results of all long-lasting projects (investment in human capital, R&D, infrastructure and so on); we consider this quantity is equal to the government spending. For simplicity, it is assumed that the product of the third sector in the amount C is completely consumed. It is known that the sum of these quantities comprises Gross Domestic Product: $Y = I + G + C$, which is the final result of activity of production subsystem. All quantities in Eqs. (1) are measured in value (money) units.

Eqs. (1) describe motion of products between sectors, which, as we know, is accompanied with motion of money that is moving in the opposite direction. There is a correspondence between fluxes of money and fluxes of products, and also as production subsystem creates real value, the bank subsystem generates corresponding amount of money. But there is no sign of the activity of bank subsystem in balance equations (1), which are written on the assumption that the money is moving without any expenses, and the banks, if present, acts free. There is no sign of money also in the expression for Gross Domestic Product. Our first task to fulfill is to make fluxes of money explicit, that is in line with the product circulation, write equations for money circulation. It allows to generalize the expression for the Gross Domestic Product and gives a solid base to understand interaction between the real and the financial sides of the economy.

In the next section, the main relations for a system, consisting of the government, a central bank, commercial banks and many customers of the commercial banks, will be formulated. We shall be urged to aggregate the variables and introduce some simplifying assumptions to make the results more transparent. Following to a work of one of the predecessor [8], we use the symbols D and B with different indexes for the amounts of deposits and debts of different customers. In accordance with the works of Keen [8,9], we will treat debt as a data record rather than a negative money. This specific assumption is very important since it implies that money used to repay debts goes into a debt account which negates an equivalent sum of debt.[1] An expression for Gross Domestic Product is formulated and discussed in Section 3. The final equations, discussed in Section 4, contain only seven variables describing money system, while it is assumed that trajectory of production output is given. The comparison with known results will be given in Conclusion.

2. The architecture of the finance system

The social production system, which is needed to maintain the existence of the human society, is considered here to be immersed in the money system that is organized and managed by a central bank and commercial banks. The situation is shown schematically in Fig. 1.

Although the money system contains many commercial banks, each with many customers, for simplicity, we consider all commercial banks together, as the only commercial bank: further, instead of many customers, we consider four groups of consumers. One can separate all accounts in the commercial banks into groups: a group of producers of main production equipment (K), a group of producers of non-material intermediate products (S), a group of producers of products for immediate consumption (C) and a group of final consumers (L). In a sense, our model is in line with a Neo-Kaleckian circuit of money, according to a classification of circuitist models by Parguez [11]. Economic actors interact with each other using money as a tool for purchase of resources, both for consumption and for production. One can assume an elementary diagram of monetary fluxes, in which only banks are accumulators of incomes and sources of expenses.

2.1. The customers of the commercial banks

At any time, when the clients of commercial banks need money, they should determine whether a financial source of possible expenses should be money from its own account or a loan from a commercial bank. The customers of commercial banks create demand for credit money, and they appear to be the basic movers of the progress of the economic system.

[1] A number of authors in the circuitist literature often assume that the repayment of debts destroys money, see Graziani [10] or Keen [8] for further information on that point.

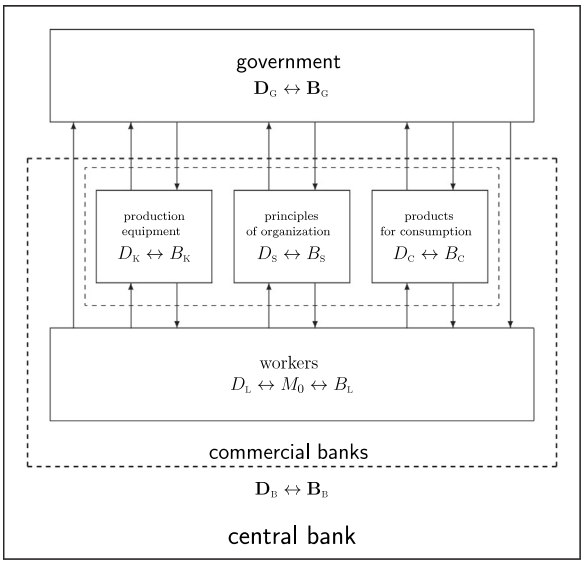

Fig. 1. The scheme of money fluxes. The central bank and the commercial banks create a money medium for activity of the economic subjects. The three sectors of the production system create all products and originate the fluxes of money between the sectors (not shown here), which determines components of Gross Domestic Product Y_{K}, Y_{S}, Y_{C}. The workers receive money in the form of wages W_{K}, W_{S}, W_{C}, W_{G}. The money is partly returning to the producers, when the workers are buying products. The government receives its part of produced value in the form of taxes $T = \theta_{\mathrm{K}}Y_{\mathrm{K}} + \theta_{\mathrm{S}}Y_{\mathrm{S}} + \theta_{\mathrm{C}}Y_{\mathrm{C}} + \theta_{\mathrm{L}}(W_{\mathrm{K}} + W_{\mathrm{S}} + W_{\mathrm{C}} + W_{\mathrm{G}})$, which in different amounts is returning to the economic subjects. To each arrow, presenting a flux of money, corresponds an arrow with the opposite directions, presenting fluxes of labor force and products. There is a bargain every time, when money is exchanged for products and labor force.

2.1.1. The producers

The balance equations (1) describe motion of products, measured by money units of value (dollar, ruble and so on); the product fluxes are accompanied by money fluxes, which are moving in the opposite direction. Considering the interaction of the production units with each other and with banks, we have to take into account expenses for bank transactions. Each production sector receives money from the sales of its product from the government, workers and all production sectors, including payments from its own sector,

$$\begin{aligned} &M_{\mathrm{K}\to\mathrm{K}} + M_{\mathrm{S}\to\mathrm{K}} + M_{\mathrm{C}\to\mathrm{K}} + Y_{\mathrm{K}},\\ &M_{\mathrm{K}\to\mathrm{S}} + M_{\mathrm{S}\to\mathrm{S}} + M_{\mathrm{C}\to\mathrm{S}} + Y_{\mathrm{S}},\\ &Y_{\mathrm{C}}. \end{aligned} \tag{2}$$

It is assumed that the quantities Y_{K}, Y_{S}, Y_{C} include both the 'pure' incomes of the sectors, that is the quantities I, G and C, and banks' incomes and can be considered to be components of Gross Domestic Product (GDP).

Simultaneously, each production sector pays (symbolized by minus signs) wages to workers and money for the products of the all sectors

$$\begin{aligned} &-M_{\mathrm{K}\to\mathrm{K}} - M_{\mathrm{K}\to\mathrm{S}} - I_{\mathrm{K}} - W_{\mathrm{K}} - \theta_{\mathrm{K}}Y_{\mathrm{K}}\\ &-M_{\mathrm{S}\to\mathrm{K}} - M_{\mathrm{S}\to\mathrm{S}} - I_{\mathrm{S}} - W_{\mathrm{S}} - \theta_{\mathrm{S}}Y_{\mathrm{S}}\\ &-M_{\mathrm{C}\to\mathrm{K}} - M_{\mathrm{C}\to\mathrm{S}} - I_{\mathrm{C}} - W_{\mathrm{C}} - \theta_{\mathrm{C}}Y_{\mathrm{C}}. \end{aligned} \tag{3}$$

Here, it is taken into account that the producers have to pay taxes to the government in the amounts $\theta_{\mathrm{K}}Y_{\mathrm{K}}$, $\theta_{\mathrm{S}}Y_{\mathrm{S}}$ and $\theta_{\mathrm{C}}Y_{\mathrm{C}}$.

Before writing the payment balance for the sectors, note that, though the receiving and payments of the money occur at one and the same time, due to a time involved for production, marketing, transportation, investment, consumption and so on, the symbols in Eqs. (2) and (3) present payments for amounts of products produced at different times. For simplicity, one can consider the symbols for intermediate products to have identical meaning, so that the payment balance (without taking into account bank services) for every production sector can be written as

$$\begin{aligned} 0 &= Y_{\mathrm{K}} + M_{\mathrm{S}\to\mathrm{K}} + M_{\mathrm{C}\to\mathrm{K}} - M_{\mathrm{K}\to\mathrm{S}} - I_{\mathrm{K}} - W_{\mathrm{K}} - \theta_{\mathrm{K}}Y_{\mathrm{K}}\\ 0 &= Y_{\mathrm{S}} + M_{\mathrm{K}\to\mathrm{S}} + M_{\mathrm{C}\to\mathrm{S}} - M_{\mathrm{S}\to\mathrm{K}} - I_{\mathrm{S}} - W_{\mathrm{S}} - \theta_{\mathrm{S}}Y_{\mathrm{S}}\\ 0 &= Y_{\mathrm{C}} - M_{\mathrm{C}\to\mathrm{K}} - M_{\mathrm{C}\to\mathrm{S}} - I_{\mathrm{C}} - W_{\mathrm{C}} - \theta_{\mathrm{C}}Y_{\mathrm{C}}. \end{aligned} \tag{4}$$

We assume, simplifying, that the commercial banks are the only source of financing of production activity, not considering a possibility of issue of shares and bonds,[2] and, consequently, we consider, that the financial states of the

[2] Distributing *shares*, production units can receive money to cover expenses directly from consumers. These primary securities are promissory notes on which emitters undertake to pay the cost of the securities and a percentage on them through a certain time and in a certain way. Money from securities is directed by the emitters to cover investment expenses, which after a while results in an additional product.

producers are defined by the amounts of deposits D and debts B (with corresponding subscripts) in the commercial banks. These quantities are obeyed to the balance equations

$$\begin{aligned}
\frac{dD_K}{dt} &= r_K D_K + Y_K + M_{S\to K} + M_{C\to K} - M_{K\to S} - I_K - W_K - \theta_K Y_K - q_K B_K + \frac{dB_K}{dt} \\
\frac{dD_S}{dt} &= r_S D_S + Y_S + M_{K\to S} + M_{C\to S} - M_{S\to K} - I_S - W_S - \theta_S Y_S - q_S B_S + \frac{dB_S}{dt} \\
\frac{dD_C}{dt} &= r_C D_C + Y_C - M_{C\to K} - M_{C\to S} - I_C - W_C - \theta_C Y_C - q_C B_C + \frac{dB_C}{dt}.
\end{aligned} \tag{5}$$

Here, we use the symbols q_K, q_S, q_C for the interest rates paid by the customers to banks for the credit, and r_K, r_S, r_C are the interest rates paid by the bank to the customers for deposits. These quantities are established by the commercial banks to adjust the quantities of deposits and debts.

To exclude the payments for intermediate products from discussion, we introduce notation for the total amount of deposits and debts of the production customers in the commercial banks

$$\begin{aligned}
D_P &= D_K + D_S + D_C, \qquad r_P D_P = r_K D_K + r_S D_S + r_C D_C, \\
B_P &= B_K + B_S + B_C, \qquad q_P B_P = q_K B_K + q_S B_S + q_C B_C.
\end{aligned}$$

Summing Eqs. (5), we get equation for the total amount of deposits and debts of the production customers in the commercial banks

$$\frac{dD_P}{dt} = r_P D_P + Y - I - W_P - \theta_P Y - q_P B_P + \frac{dB_P}{dt}. \tag{6}$$

Here a taxation rate for the productive actors θ_P is introduced as a weighted sum coming from θ_K, θ_S, θ_C.

2.1.2. The consumers

The financial state of the consumers is determined by the amounts of deposits and debts, D_L and B_L, in the commercial banks. In addition, they are holders of paper money in the amount M_o—this is cash money, which is in circulation at the moment. The consumers use their money and possible loans to acquire products, so that the balance equations for the consumers can be written as

$$\frac{dD_L}{dt} + \frac{dM_0}{dt} = r_L D_L + W - C - \theta_L W - q_L B_L + E_0 + \frac{dB_L}{dt}, \tag{7}$$

where $W = W_K + W_S + W_C + W_G$ is a flux of money to workers in the form of wages, which are received from the production sectors (W_K, W_S, W_C) and the government (W_G). The bank emission of paper money E_0, generally speaking, is not equal to a change of the amount of circulating paper money, but if we do not consider processes of transformation of paper money in non-cash and on the contrary, we have to suppose that $E_0 = \frac{dM_0}{dt}$, what we accept in the next. The consumers pay money in the amount C to the third sector for the consumption products, which were created some time ago, and to the government in the form of taxes, $\theta_L W$, so that, in equilibrium situation, one has

$$C = (1 - \theta_L) W.$$

In a reality, a situation can be somewhat more complex; part of wages can be used for purchase shares of the enterprises, that is, for investment in various sectors, which we do not consider here.

The right-hand sides of Eqs. (5) and (7) contain payments to and by the commercial banks. The banks ask the interest rates q_P, q_L for debts and give the interest rates r_P, r_L to customers for their deposits. So, as there is a payment for debts, customers try to reduce quantity of debts as far as possible and to keep some money on the depositary accounts in commercial banks.

2.1.3. The united customer of commercial bank

For simplicity of consideration, we unite deposits and credits of producers and consumers in commercial bank in united quantities and introduce new variables according to the rules

$$\begin{aligned}
D &= D_P + D_L, \qquad r = (r_P D_P + r_L D_L)/(D_P + D_L), \\
B &= B_P + B_L, \qquad q = (q_P B_P + q_L B_L)/(B_P + B_L).
\end{aligned}$$

The balance equation for these variables is recorded as the sum of Eqs. (6) and (7), so that

$$\frac{dD}{dt} = rD - qB + Y - I - C + W_G - T + A_0 - \kappa(D + M_0) + \frac{dB}{dt}. \tag{8}$$

Above the sum, the equation contains two additional terms. To take into account the commission fees for the transactions between subjects (which have been omitted at aggregation of variables), and various gathering for carrying out of operations, the quantity proportional to the amount of money $D + M_0$ is installed into Eq. (8). This equation is also added by the quantity

A_0, which represents a possible flux of money into the accounts of clients from external sources. The quantities of such type should be present also at Eqs. (6) and (7), but at summation additional quantities reduce to the additives, specified in Eq. (8). The factor of proportionality κ represents an assessment of efforts on maintenance of one monetary unit under circulation and is a characteristic of the system.

Also as Eqs. (6) and (7), Eq. (8) contains payments for services of commercial banks. Banks establish norms of payments for credits and deposits of clients q and r which are, generally speaking, functions of deposits and credits; they are established by commercial banks from the requirement to receive some profit on bank operations (see the following section).

2.2. Commercial bank as a customer of the central bank and a supplier of credit money

We consider the commercial banks are intermediaries in the interaction among the production sectors and consumers within the economic system. The money deposits with commercial banks and loans from the banks are the means to organize and facilitate the interaction of economic actors. The commercial banks are supported by central bank and are motivated by desire to get profit from the operations with the customers.

2.2.1. The balance of commercial bank

Considering all commercial banks as the only bank, we assume that the bank has customers' deposits and debts in amounts D and B, correspondingly. One can assume that the commercial bank has the only account with the central bank D_{B}, on which it holds all its reserve, including the amount of mandatory deposit of the commercial bank ξD, where ξ is a norm of the mandatory reserve deposit that is set up by the central bank. The commercial banks have also a debt B_{B} to the central bank.

The state of the commercial bank is determined by its actives: K_{KB}, $D_{\mathrm{B}} - \xi D$, B and passives: B_{B}, D. The income of the bank, neglecting the income from the bank's capital K_{KB} and any other operations, can be written as

$$r_{\mathrm{B}}(D_{\mathrm{B}} - \xi D) - q_{\mathrm{B}}B_{\mathrm{B}} + qB - rD. \tag{9}$$

The central bank fixes the interest rate given by the banks for its deposit r_{B} and the interest rates q_{B} asked by the bank for debts of the commercial bank, and the commercial bank sets the interest rate given by banks (to customers) for their deposits r and the interest rates q asked by banks for debts of its clients. In any case, it is expected, that value of q with any index will appear greater, than value of r with an appropriating index. Usually the central bank does not pay for mandatory deposits of commercial banks and sets up a high level of the refinancing rate q_{B}. The norm of the mandatory reserve deposit ξ and the refinancing rate q_{B} are considered as main regulators of the amount of non-paper money.

The expression (9) ought to contain commission payments and other gathering, which ought to be included in the sum $\kappa(D + M_0)$, but we assume that this income completely goes on employee wages and current maintenance of the bank.

The deposit D_{B} changes due to its income (9), the changes of the debt to the central bank B_{B} and operations with the customers of commercial banks, so that the balance equation can be written as

$$\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}t} = r_{\mathrm{B}}(D_{\mathrm{B}} - \xi D) - q_{\mathrm{B}}B_{\mathrm{B}} + qB - rD + \frac{\mathrm{d}B_{\mathrm{B}}}{\mathrm{d}t} + \frac{\mathrm{d}(D - B)}{\mathrm{d}t} + A_{\mathrm{C}}. \tag{10}$$

Here we assume, that the stream of money into the account of commercial bank from external sources A_{C} is possible.

By virtue of Eq. (8), Eq. (10) can be rewritten in the form

$$\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}t} = r_{\mathrm{B}}(D_{\mathrm{B}} - \xi D) - q_{\mathrm{B}}B_{\mathrm{B}} + Y - I - C + W_{\mathrm{G}} - T - \kappa(D + M_0) + A_0 + E_{\mathrm{C}}. \tag{11}$$

Here we introduce a symbol for emissions of credit money, taking into account the external flux of money,

$$E_{\mathrm{C}} = A_{\mathrm{C}} + \frac{\mathrm{d}B_{\mathrm{B}}}{\mathrm{d}t}. \tag{12}$$

2.2.2. Mechanism of creation of credit money

Primary activity of commercial banks is connected with crediting the clients. Usually the aggregate amounts of loans B and credits D appear to be greater than available banks' reserves $D_{\mathrm{B}} - \xi D$; commercial banks create credit money out of nothing—the evidence that ought to be taken into account. According to Werner [12], there are three main theories of banking activity. A first theory states that banks are merely intermediaries like other non-bank financial institutions, collecting deposits that are then lent out. A second, the fractional reserve theory of banking generalizes the first theory and asserts that banks collectively end up creating money through systemic interaction. A third theory, which is proposed by Werner and supported by researches of Bank of England [13], maintains that each individual bank has the power to create money 'out of nothing' and does so, when it extends credit (the credit creation theory of banking).

According to the modern representation of banking activity [12,13], the individual commercial banks, while providing loans, do not feel any direct constraints from the central bank; the reserve account in the central bank remains untouched [12]. To increase the profit, the commercial banks are motivated to produce more credits to their customers,

but increase in credits B to the customers apparently meets some restrictions: the operation ought to be acceptable both for the commercial bank and the customers. But ultimate constraint for money creation, as the researchers [13] assert on p. 4, is monetary policy of the central bank. However, it remains unclear what instruments the central bank does use to influence on the money creation, if each individual commercial bank does not pay any attention on the amount of its reserve. This is a problem in the credit creation theory of banking that ought to be cleared by its proponents.

In contrast to it, the fractional reserve theory, in which the restrictions are connected with available reserves, is developed in all details. According to the well-known mechanism of multiplication (see, for example, Samuelson and Nordhaus [14], p. 240), enlargement of amount of available money $\Delta A = \Delta (D_{\mathrm{B}} - \xi D)$ from the central bank and deposits allows the commercial bank to lend $(1 - \xi)\Delta A$ to clients and other banks, whereas the part $\xi \Delta A$ of the total amount must be reserved in the central bank. The banks use the amount $(1 - \xi)\Delta A$ for further lending, leaving the part $\xi (1 - \xi)\Delta A$ of the amount in the central bank as a reserve. The process is continuing, so that the banks are creating money on the customers' deposits in the total amount

$$\Delta A + (1 - \xi)\Delta A + (1 - \xi)^2 \Delta A + \cdots = \frac{1}{\xi}\Delta A,$$

and one can write the relation

$$\frac{\mathrm{d}B}{\mathrm{d}t} \leq \frac{1}{\xi}\frac{\mathrm{d}\,(D_{\mathrm{B}} - \xi D)}{\mathrm{d}t}. \tag{13}$$

This equation defines restriction on release of credit money. The quantity $1/\xi$ appears the multiplicator showing a possible increase of credit money. Apparently, the mechanism of multiplication works at large number of commercial banks and Eq. (13) is valid for the entity of banks, not for individual bank.

2.2.3. Is it possible to reconcile the two approaches?

There are apparently some discrepancies in the explanation of credit money creation by the existing theories. One can get such an impression that the fractional reserve theory and the credit creation theory describe the money creation from different points of view: the first one operates with aggregate quantities (macroeconomic approach), while the second considers variables and concepts describing individual banks (microeconomic approach). It means that each individual commercial bank acts, as it is described by Werner [12], freely and its credits can even overpass its reserves, but loans issued by all commercial banks collectively cannot be greater than their aggregate reserve. One can think that, after considering the proper role of the central bank and aggregation, the constraint in the credit creation theory could be formulated, as some relation among the aggregate quantities that could be similar to inequality (13).

If one considers aspiration of commercial banks to expand the credit and some rationality of their behavior, it is possible to expect that the inequality (13) trend to become equality. Anyway, it is possible to introduce an effective quantity $\xi^* > \xi$, at which the parity (13) is read as equality. The ratio ξ/ξ^* shows the breadth (depth) of propagation of credit money after some permutation. Then, it follows, from a relation (13) (at constant ξ), a parity between derivatives of the quantities

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{1}{\xi^*}\frac{\mathrm{d}D_{\mathrm{B}}}{\mathrm{d}t} - \frac{\mathrm{d}D}{\mathrm{d}t}. \tag{14}$$

At the fixed value ξ^*, Eq. (14) defines a restrictive condition on a possibility of banks to increase credits. There are, apparently, some other restrictive parities on the quantity of loans to clients of commercial bank B, imposed by Eqs. (8) and (11). Within the limits of these restrictions, commercial banks define amount of the loans B_{B} from the central bank, and clients of commercial banks define quantities of the deposits D and debts B. At this, the bank requires a quantity of a seed capital K_{KB} to start the operations.

2.3. The government as a customer of the central bank and the central bank as a producer of paper money

The institution that is crucial in organization of money circulation in a society, is a central bank, which is a bank of the commercial banks and the bank of the government. The activity of the central bank is closely connected with the activity of the government and is based on the credit to the government and the central bank's asset. To organize the money circulation, the central bank issues money in form of paper notes (coins) and credits to commercial banks. The central bank creates fiat money that sets up a scale of value.

2.3.1. The central bank

It is supposed that the central bank is established for the aim to organize, together with the government, circulation of money in the system. Besides that, the central bank accounts the incomes and expenses of the government, financial state of which is fixed, in an elementary case, by two quantities: amount of available money D_{G} and debts B_{G}. The state of the central bank is fixed by its actives: K_{CB}, B_{G}, B_{B} and passives: D_{G}, D_{B}, M_0. On the disposal of the central bank and the government, there is a profit from the basic activity

$$q_{\mathrm{G}} B_{\mathrm{G}} + q_{\mathrm{B}} B_{\mathrm{B}} - r_{\mathrm{G}} D_{\mathrm{G}} - r_{\mathrm{B}}(D_{\mathrm{B}} - \xi D), \tag{15}$$

where $q_B B_B$ is the payment of the commercial banks for use of credits of the central bank, q_B is a refinancing rate. The rate of interest for debts and deposits of the government, q_G and r_G, in Eq. (15) are specified by agreement of the central bank with the government. Due to its close relationship with the government, the central bank does not intend to get any profit from the service to the government.

2.3.2. The balance of the government

The government bothers to have on its disposal enough money for financing national projects G and salary payments to the civil servants W_G. The main account of the government with the central bank presents the governmental budget and reflects motion of money to and from the government. The incoming fluxes of money include taxes (and other incomes) T into the budget, which are the payments from the producers and consumers

$$T = \theta_P Y + \theta_L W,$$

where $Y = Y_K + Y_S + Y_C$ is the gross domestic product (GDP) with contribution from the three production sectors and $W = W_K + W_S + W_C + W_G$ is the total amount of wages paid to workers and civil servants. The government supervises norms of the taxation θ_P and θ_L to provide expenses of the government G, which represents investments in various national projects, and wage payments to the civil servants W_G.

The profit of the central bank (15), minus expenses on the organization of the circulation of money, comprises the amount of money on the government's disposal D_G that obeys the balance equation

$$\frac{dD_G}{dt} = q_B B_B - r_B(D_B - \xi D) + T - G - W_G + \frac{dB_G}{dt} + A_G + E_0. \tag{16}$$

For financing the activity, the government can let out paper money E_0 and (or) address to creditors $\frac{dB_G}{dt}$. It is supposed also, that external loans – a flux of money A_G – from external sources are possible, so that, alongside with a designation for emission of paper money, a new designation is introduced

$$E_0 = \frac{dM_0}{dt}, \qquad E_G = A_C + \frac{dB_G}{dt}. \tag{17}$$

Let us notice, that at more detailed analysis, it is necessary to consider, that, if the government pays money in the moment t, it receives taxes from the earlier activity. The loan is necessary to provide the governmental expenses G and W_G, so that it is possible to think, that the amount of the loan is connected with the amount of expenses.

3. Money circulation and production

The assumption about composition and architecture of the system, consisting of the government, the central bank, the commercial banks and many production and consumption units, allow us to describe the situation. The economic subjects are interacting with each other by means of money fluxes. To create money, the central bank issues the coins and paper money in the amount of M_o and credits the commercial banks in the amount of B_B. The sum of the issued paper and non-paper money, $M_o + D_B$ is called the *monetary base*. The credits of the central bank B_B to the commercial banks provide an opportunity to credit the producers and consumers, thus creating deposits D, which can be called non-paper money. The non-paper money can be transformed into paper money and, on the contrary, the paper money can be transformed into non-paper money, so that the characteristic quantity is the sum of all deposits in commercial banks D and paper money M_o. The total is called the *monetary mass*, for which a conventional symbol $M_2 = M_o + D$ is used. The process of introducing and circulating money is described by the equations, formulated in the previous section, and our task now is to estimate the amounts of both paper and non-paper money needed for the proper functioning of the production system.

3.1. The program of development of production–consumption system

In the 'basement' of the program of economic activity, one can find apparently the real consumption and production. John Maynard Keynes wrote in his *Treatise on Money* that '[h]uman effort and human consumption are the ultimate matters from which alone economic transactions are capable of deriving any significance; and all other forms of expenditure only acquire importance from their having some relationship, sooner or later, to the effort of producers or to the expenditure of consumers' [15], p. 120-1. In the basis of any program of economic development, it is possible to find, apparently, the program of consumption and production. It is impossible to exclude, certainly, influence of monetary circulation on production subsector of a national economy, but, nevertheless, in considered approximation we consider, that the industrial sector develops under its own laws.

It is natural to believe, that, by studying the actual situation, the producers, the government and the consumers can adapt their programs of development and expenditure, which can be described by means of the time-dependent rates of growth

as

$$\frac{\mathrm{d}\underline{I}}{\mathrm{d}t} = \sigma_{\mathrm{I}}\underline{I}, \qquad \frac{\mathrm{d}\underline{G}}{\mathrm{d}t} = \sigma_{\mathrm{G}}\underline{G}, \qquad \frac{\mathrm{d}\underline{C}}{\mathrm{d}t} = \sigma_{\mathrm{C}}\underline{C},$$
$$\frac{\mathrm{d}\underline{W_{\mathrm{P}}}}{\mathrm{d}t} = \psi_{\mathrm{P}}\underline{W_{\mathrm{P}}}, \qquad \frac{\mathrm{d}\underline{W_{\mathrm{G}}}}{\mathrm{d}t} = \psi_{\mathrm{G}}\underline{W_{\mathrm{G}}}. \tag{18}$$

Let us notice, that at planning they are interested, as a rule, in the growth of actual output of production and the growth of the actual wages, so that all quantities in the Eqs. (18), designated by underlining, are estimated by monetary unit of constant purchasing capacity. In a reality, purchasing capacity of monetary unit can change, that usually describe introduction of price indexes. Certainly, the money supply is distributed non-uniformly on sectors of production, one sector may have a lot of money, while the others—less, so that the usage of several price indexes is necessary, but for simplicity, we use the only price index ρ, which is introduced by a relation

$$I + G + C = \rho\,(\underline{I} + \underline{G} + \underline{C}). \tag{19}$$

Symbols without the underlining represent assessments of the quantities with the current monetary units.

The rates of growth $\sigma_{\mathrm{I}}, \sigma_{\mathrm{G}}, \sigma_{\mathrm{C}}, \psi_{\mathrm{W}}, \psi_{\mathrm{G}}$ in Eqs. (18) are, generally speaking, functions of time which are estimated or appointed by operating subjects. Average value of the rates of growth can be expressed through the quantities measured by arbitrary monetary unit,

$$\sigma = \frac{1}{I + G + C}(\sigma_{\mathrm{I}} I + \sigma_{\mathrm{G}} G + \sigma_{\mathrm{C}} C),$$
$$\psi = \frac{1}{W_{\mathrm{P}} + W_{\mathrm{G}}}(\psi_{\mathrm{P}} W_{\mathrm{P}} + \psi_{\mathrm{G}} W_{\mathrm{G}}). \tag{20}$$

The government, alongside with the expenses connected with maintenance of the general projects G and wages payment to the civil servants W_{G}, plans receiving the income in the form of taxes

$$T = \theta_{\mathrm{P}} Y + \theta_{\mathrm{L}}(W_{\mathrm{P}} + W_{\mathrm{G}}). \tag{21}$$

With a view of balancing the state budget, the government establishes norms of taxes θ_{P} and θ_{L}.

In this approach, we focus only on the monetary circulation meaning that I, G and C are assumed to be given as a function of time to allow considering monetary fluxes in the system. However, a more detailed definition of these functions would imply a specific theory of production for the three involved sectors (this objective is out of scope of this paper).

3.2. The gross domestic product

An expression for the important characteristics of the system – the Gross Domestic Product, Y – can be obtained, when Eqs. (8), (10) and (16) have been aggregated

$$Y = I + G + C - (A_0 + A_{\mathrm{C}} + A_{\mathrm{G}}) + \kappa(D + M_0) - \frac{\mathrm{d}M_0}{\mathrm{d}t} + \frac{\mathrm{d}(D_{\mathrm{B}} - B_{\mathrm{B}})}{\mathrm{d}t} + \frac{\mathrm{d}(D_{\mathrm{G}} - B_{\mathrm{G}})}{\mathrm{d}t}. \tag{22}$$

This formula is, apparently, generalization of conventional expression of the Gross Domestic Product Y as the sums of assessments of investments I, the governmental expenses G and immediate consumption C. In addition to these quantities, the expression (22) contains export of money by clients, commercial banks and the central bank, $-(A_0+A_{\mathrm{C}}+A_{\mathrm{G}})$, accordingly. The quantity $\kappa(D + M_0)$ represents an assessment of efforts on maintenance of the circulation of money in amount $M_0 + D$ (the coefficient κ represents an assessment of efforts on maintenance of the circulation of one monetary unit). The last two terms in expression (22) show, that the part of the added value is preserved (with a sign plus) on accounts of the central bank.

The first terms of expression (22) present average quantities, while the three terms represent pulsating quantities, which should be anyhow averaged to exclude the pulsating parts from consideration. It is possible to believe, that after averaging the Gross Domestic Product can be presented as

$$Y = I + G + C - (A_0 + A_{\mathrm{C}} + A_{\mathrm{G}}) + \kappa(D + M_0). \tag{23}$$

Last term in this expression includes all expenses on emission of bank notes and the organizations of monetary circulation; we can assume, that average value of the two last terms in relation (22) is included also in this assessment.

The relation (23) can be presented in other form, if one introduces a symbol R for the ratio of an assessment of services of bank system to the pure output of production system.

$$Y = (1 + R)(I + G + C) - (A_0 + A_{\mathrm{C}} + A_{\mathrm{G}}). \tag{24}$$

It is possible to believe, that the quantity R changes slowly and can be considered as a characteristic of the system. Apparently, this quantity determines efficiency of a social production: the more the size R, the more expenses for maintenance of monetary circulation and the less the efficiency of the system.

In this way, we have integrated the banks profit into the national income and, therefore, into the money credit process. The presence of banking sector in GDP generated many debates in the circuitist literature, which often fail to take properly into account the banks profit: "interest payments on loans made from firms to banks are not accounted as part of national income and simply disappear, instead of being treated as a possible source of demand for goods and\or financial assets" [16], p. 1. According to Rochon [17], p. 125, "The existence of monetary profits at the macroeconomic (aggregate) level has always been a conundrum for theoreticians of the monetary circuit...".

3.3. The price index and the quantitative theory of money

Parities (23) and (24) allow to establish a relation between total amount of money $M_2 = M_o + D$, and production output. Being limited to consideration of the average quantities, on the basis of the specified equations, it is possible to record the relation

$$M_o + D = \frac{R}{\kappa}(I + G + C) = \rho\,\frac{R}{\kappa}(\underline{I} + \underline{G} + \underline{C}). \tag{25}$$

Here, relation (20), defining a price index ρ, is used also.

Eq. (25) defines a total amount of money $M_2 = M_o + D$, circulating in the system at a given value of the real production output estimated in arbitrary, current or constants, monetary units. This equation establishes "physical" content of monetary unit, which is not known beforehand. The factor of proportionality R/κ, also as the quantities R and κ separately, is the characteristic of the production subsystem, which, it is possible to believe, changes slowly.[3]

The price index ρ is one of the main quantities describing monetary circulation, its constancy (stability) is a condition and a certificate of normal functioning of a national economy; values of the index exceeding unity provide some temporary income to the government, however values $\rho \gg 1$ destroy economic interactions. For the analysis it is convenient to use expression for the rate of growth of the price index. Differentiating relation (25), we find

$$\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{M_o + D}\left(\frac{\mathrm{d}M_0}{\mathrm{d}t} + \frac{\mathrm{d}D}{\mathrm{d}t}\right) - \frac{\kappa}{R}\frac{\mathrm{d}(R/\kappa)}{\mathrm{d}t} - \frac{\sigma_I I + \sigma_G G + \sigma_C C}{I + G + C}. \tag{26}$$

The recorded relation contains three terms, connected with production output variation, evolution of properties of the system and monetary fluxes, and defines conditions of the growth of production at the prospective. Generally the rate of variation of the price index depends on a state of the system, the rates of growth of production output and, certainly, from the fluxes of money emerging within the system.

4. Dynamics of the system

4.1. The system of evolutionary equations

The basis of the system of evolutionary equations comprises the balance equations discussed in the second Section. Eqs. (8), (11), (14) and (16) are connecting the state variable of the five interacting economic subjects, each one of which possesses the certain financial actives and has its own tactics of behavior. We are considering the closed system, taking into account influence of external factors as it is fixed at the formulation of balance equations. By consideration of a financial system in this (quasi-closed) approximation, the quantities E_0, E_C and E_G represent now the sources of money, in which the external fluxes A_0, A_C, A_G, that should be set independently, are included. Addressing to the specified relations (8), (11), (14), (16) and uniting Eqs. (8) and (14), we record system of the evolutionary equations in the form of convenient for the analysis

$$\begin{aligned}
\frac{\mathrm{d}D_B}{\mathrm{d}t} &= r_B(D_B - \xi D) - q_B B_B - \Delta - A_C - A_G + E_C,\\
\frac{\mathrm{d}D}{\mathrm{d}t} &= \frac{1}{2}(rD - qB) + \frac{1}{2\xi^*}\left[r_B(D_B - \xi D) - q_B B_B\right] - \frac{1+\xi^*}{2\xi^*}\left[\Delta + A_C + A_G\right] + \frac{1}{2\xi^*}E_C,\\
\frac{\mathrm{d}D_G}{\mathrm{d}t} &= q_B B_B - r_B(D_B - \xi D) + \Delta + E_G + E_0.\\
\frac{\mathrm{d}B}{\mathrm{d}t} &= -\frac{1}{2}(rD - qB) + \frac{1}{2\xi^*}\left[r_B(D_B - \xi D) - q_B B_B\right] - \frac{1-\xi^*}{2\xi^*}\left[\Delta + A_C + A_G\right] + \frac{1}{2\xi^*}E_C,\\
\frac{\mathrm{d}M_0}{\mathrm{d}t} &= E_0, \qquad \frac{\mathrm{d}B_B}{\mathrm{d}t} = E_C - A_C, \qquad \frac{\mathrm{d}B_G}{\mathrm{d}t} = E_G - A_G.
\end{aligned} \tag{27}$$

[3] The relation (25) is known as expression so-called *the quantity theory of money* [18], in which the quantity R/κ has been interpreted as average time of the rotation of money (time from manufacture before consumption of an end-product). This relation is also known as Fisher's relation, though, according to Harrod [19], p. 26 this law was classically exposed in the report of the British Bullion Committee in year 1810. Moreover, Harrod notes: 'Of course, the Bullion Committee did not invent the quantity theory. Traces of it may be found in writers dating back for centuries before that'.

While recording the equations, definition (24) for output Y is used. A symbol for excess of incomes of the government over expenses—proficit of the budget is introduced

$$\Delta = T - G - W_G.$$

The quantity in the right part of the relation is connected with the production program (18) and (21). An additional condition is also non-negativity of the incomes of commercial and central banks (9) and (15).

The global characteristics of the production-distributive system κ and R ought to be specified. The central bank establishes norm of mandatory deposit ξ that does not coincide with its effective value ξ^*, determining the expansion of credit money. These quantities should be set. The central bank establishes also norms of payments r_B, q_B for deposit and credit of the commercial bank, which, in its turn, defines norms of payments for deposits and credits, r and q, of the clients. The parameters, established by the government, the central bank and commercial bank, are not constant, but depend on the situation and ways of behavior of economic subjects. Apparently, various models of behavior of agents are possible, and it is necessary to analyze the actual situation to formulate appropriating dependences, which are neglected further.

The system of the Eqs. (27) describes behavior of seven variables D, D_B, D_G, B, B_B, B_G and M_0 at preset values of the listed parameters, and, eventually, evolution of financial variables is determined by the sources of money of a type E_0, E_C, E_G, which includes the external fluxes A_C and A_G. It is remarkable, that the external stream A_0, also as the real production defined the quantities I, G, C, W_P and W_G, do not enter into the system of the Eqs. (27). The financial subsystem is developing autonomously, the linkage with the real production is being established through the sources (emission) of money E_0, E_C, E_G (see further Section 4.3).

4.2. The steady-state situation

The equations of evolution determine a trajectory of the system at the characteristics of system defined by the activity of the ensemble of participants of actual processes of consumption-production. It is convenient to consider, first of all, a stationary case, when it is possible to consider the all characteristics, as well as all system's variables, to have constant values. Economists call such situations equilibrium, but, from the thermodynamic point of view, the considered system is in a stationary, nonequilibrium state. The state of the production-monetary system is defined in this case only by balance equations, without any additional assumptions.

4.2.1. The steady-state situation

We consider the established state of a financial system when all variables defined by the Eqs. (27) are constant, however the rates of growth of the production subsystem can be any. Provided that $E_0 = 0$, but the external fluxes A_0, $A_C = E_C$, $A_G = E_G$ are possible, and the equations of evolution (27) reduce to the algebraic equations

$$\begin{aligned} 0 &= r_B(D_B - \xi D) - q_B B_B - \Delta - A_G, \\ 0 &= rD - qB - \Delta - A_C - A_G. \end{aligned} \tag{28}$$

The system of equations does not define a unique point in phase space: the number of variables is more than number of the equations. Eqs. (28) show that, in the established situation, two variables only from a set D, D_B, B and B_B ought to be considered independent. When proficit of the budget is excluded from Eqs. (28), one has a relation

$$A_C = rD - qB + q_B B_B - r_B(D_B - \xi D). \tag{29}$$

Eqs. (28) can be presented in the form of

$$\begin{aligned} D_B &= \xi D + \frac{q_B}{r_B}(B_B - B_B^0), \qquad B_B^0 = \frac{1}{q_B}(-\Delta - A_G), \\ D &= \frac{q}{r}(B - B^0), \qquad B^0 = \frac{1}{q}(-\Delta - A_C - A_G). \end{aligned} \tag{30}$$

These equations define quantities of deposits in steady state D and D_B, as functions of loans B and B_B. The amounts of deposits should be considered non-negative, so that Eqs. (30) define amounts of loans, which are necessary that the commercial bank could start to function. The amounts of loans with zeroes B^0 and B_B^0 can be interpreted, accordingly, as a minimum quantity of the capital of commercial bank and the loan from the central bank, which allow bank system to start action.

4.2.2. The measure of propagation of credit money

Eq. (14) allows to find expression for effective quantity ξ^* in a stationary case. Really, this equation is followed (at constant value ξ^*) by an expression for the quantity of money, which is on the disposal of commercial bank

$$D_B = \xi^*(B + D) + const. \tag{31}$$

Considering this equation at zero deposits, one can define a constant by means of the first line from system (30), so that the equation now can be expressed in the form of

$$D_B = \xi^*(D + B - B^0).$$

This equation defines stationary value of the quantity ξ^*

$$\xi^* = \frac{D_B}{rD + B - B^0}. \tag{32}$$

Let us remind, that the relations recorded here are valid for the steady-state situation or, as economists speak, for a situation of equilibrium.

4.3. Rules of emission

The trajectory of evolution of the monetary system, described by the system of equations (27), is essentially determined by values of emission of paper and credit money E_C, E_o, E_G, including the fluxes of money in system A_0, A_C and A_G. The listed quantities are being set by the economic subjects (clients of banks, commercial and central banks, as well as the government) after estimation of the situation, in which the subjects appear to be. In particular, the external fluxes depend on conditions of managing inside and outside of the system; in our approximation we consider values A_0, A_C and A_G to be given. However, the internal emission of paper and credit money are being done by economic subjects, and it is possible to formulate some rules for this emission.

It is possible to believe, that the central bank monitors progress of real production and supervises its accounts, aiming that the total amount of money in the system $D + M_0$ would correspond to the real production output in the sense defined by relation (25). It means, that the central bank watches, that the rate of growth of the price index (inflation), defined by Eq. (26), would not be too great or even was equal to zero. To reach the desirable correspondence, the central bank considers the deposits of the government and commercial bank, D_G and D_B, would increase according to progress of production subsector; in the simplest case, with the rate of growth of production output σ. Then, the first and the third equations from system (27) define emission of paper and credit money

$$\begin{aligned} E_0 + E_G &= \sigma D_G - q_B B_B + r_B(D_B - \xi D) - \Delta, \\ E_C &= \sigma D_B - r_B(D_B - \xi D) + q_B B_B + \Delta + A_C + A_G. \end{aligned} \tag{33}$$

The quantity E_o and E_G are defined by the government and the central bank, proceeding from the necessity to provide additional financing of the budget area and government projects. The emission of credit money is defined by joint actions of the central and commercial banks.

The lack of money in the budget can be immediately compensated by release of paper money E_0 and obligation E_G. However, the emission cannot proceed long time, therefore the government could undertake measures on increase in tax revenues, increasing rates of taxes θ_P and θ_L to receive the desirable income

$$T = \theta_P Y + \theta_L(W_P + W_G),$$

for example, by rules

$$\frac{d\theta_P}{dt} = -n_P \Delta, \qquad \frac{d\theta_L}{dt} = -n_L \Delta, \tag{34}$$

where factors n_P and n_L define the rate of change of the tax rates. These quantities, as well as the norms of taxes θ_P and θ_L, are fixed by the central bodies: the government and the central bank, which start with their intentions and assessment of the situation in the production subsector.

There is no automatic causal relationship between development of production and evolution of monetary system; such linkages are being established by hands of workers of the central bank after some analysis. Apparently, rules of emission can be formulated on the basis of various requirements, for example, specifying small inflation that is a preferential version for the existing governments, so as it provides the additional income into the budget. To support the production development, the central organizations should define the program of issue of credit and paper money. The simple reasons stated above are based on the assumptions, that the government and the central bank in their activity are guided only by the interests of creation of favorable environment for development of national production. Unfortunately, it is not always true: if a national monetary system is strongly integrated into the World environment, when economic subjects can interact directly with the external agents, there appears additional reasons connected with the necessity to provide favorable (or adverse) interoperability with external agents. Additional conditions can lead to contradictions with national interests and the big art is required from the bank managers to maneuver among controversial requirements.

5. Conclusion

We have presented a simplest model of money circulation in the productive–distributive system, which is believed can be useful to investigate the main features of interaction between production and money circulation. A specific feature of our approach to the description of monetary circulation is introduction and use of fundamental characteristics of the system, among which: the ratio of system's efficiency R, the expenses for maintenance of one monetary unit κ and a measure of propagation of credit money ξ^*. The ratio κ/R appears to be 'the velocity of circulation of money' in the known 'quantity

theory of money'. The considered scheme represents the most general features of functioning of any national economic system and shows a way, by which two sides of social production: motion of real things and motion of money can be integrated into the theory of macroeconomics.

It was shown that production and financial subsystems are developing independently; their interrelation is defined by a relation between the rate of growth of production and emission of paper and credit money, which is established by managers of the central bank, being based on the analysis of the situation. While estimating the rates of real growth, humans can make involuntary mistakes, and correspondence of monetary fluxes to the fluxes of products can be disturbed. In fact, neither the central bank, nor commercial banks can directly adjust quantity of both paper and non-paper money, and the purchasing capacity of the unit of money (ruble, dollar, euro and so on) can change in due course, as it is observed in reality. The decision of the central bank on emission of money defines 'a physical content' of monetary unit and the price index in conformity with a fundamental relation (25) of 'the quantity theory of money'.

Let us notice once more, that our description is rather rough: we used highly aggregated variables, which, nevertheless, provides the way to describe the monetary circulation and establish some laws of evolution of the system. The real situation is, of coarse, more complex: the results of the theory depend on the specific assumptions on the system architecture and preferences of the process participants.

Acknowledgments

The authors are grateful to the three anonymous reviewers for their constructive critiques that helped improve the quality of this contribution. One of the authors (V.N.P.) is grateful to the Russian Science Foundation for the support (Project N^o 14-11-00634).

References

[1] W. Godley, M. Lavoie, Monetary Macroeconomics: An Integrated Approach to Credit, Money, Income, Production and Wealth, Palgrave Macmillan, Basingstoke, 2007.
[2] G. Deleplace, E. Nell (Eds.), Money in Motion: The Post Keynesian and the Circuilationist Approaches, Macmillan, London, 1996.
[3] G. Fontana, R. Realfonzo (Eds.), The Monetary Theory of Production: Tradition and Perspectives, Palgrave Macmillan, 2005.
[4] V.M. Yakovenko, J. Barkley Rosser, Statistical mechanics of money, wealth and income, Rev. Modern Phys. 81 (2009) 1703–1725.
[5] B.K. Chakrabarti, A. Chakraborti, S.R. Chakravarty, A. Chatterjee, Econophysics of Income and Wealth Distributions, Cambridge University Press, Cambridge, 2013.
[6] V.N. Pokrovskii, Econodynamics. The Theory of Social Production, Springer, Dordrecht-Heidelberg-London-New York, 2011, http://www.springer.com/physics/complexity/book/978-94-007-2095-4.
[7] W.W. Leontiev, Input-Output Economics, second ed., Oxford University Press, New York, Oxford, 1986.
[8] S. Keen, The dynamics of the monetary circuit, in: S. Rossi, J.-F. Ponsot (Eds.), The Political Economy of Monetary Circuits: Tradition and Change, Palgrave Macmillan, London, 2009, pp. 161–187.
[9] S. Keen, Keynes's 'revolving fund of finance' and transactions in the circuit, in: R. Wray, M. Forstarter (Eds.), Keynes and Macroeconomics after 70 Years, Edwar Elgar, Aldershot, 2008.
[10] A. Graziani, Money as purchasing power and money as a stock of wealth in a Keynesian economic thought, in: G. Deleplace, E. Nell (Eds.), Money in Motion: The Post Keynesian and the Circuilationist Approaches, Macmillan, London, 1996.
[11] A. Parguez, The solution of the paradox profits, in: R. Arena, N. Salvadori (Eds.), Money, Credit and the Role of State, Ashgate Press, Burlington, 2003.
[12] R.A. Werner, Can banks individually create money out of nothing? - The theories and the empirical evidence, Int. Rev. Financ. Anal. 36 (2014) 1–19.
[13] M. McLeay, A. Radia, R. Thomas, Money creation in the modern economy, Bank Engl. Quart. Bull. 1 (2014) 14–27. http://www.bankofengland.co.uk/publications/Documents/quarterlybulletin/2014/qb14q102.pdf.
[14] P. Samuelson, W. Nordhaus, Economics, thirteenth ed., McGraw-Hill Book Company, New York et cetera, 1989.
[15] J.M. Keynes, A Treatise on Money, Macmillan, London, 1971, Reprinted in The Collected Writings of John Maynard Keynes, Volumes V and VI, London and Basingstoke: Macmillan for the Royal Economic Society.
[16] G. Zezza, Some Simple, Consistent Models of the Monetary Circuit, Jerome Levy Economics Institute of Bard College, Working Paper no 1405 (2004).
[17] L.-P. Rochon, The existence of monetary profits within the monetary circuit, in: G. Fontana, R. Realfonzo (Eds.), The Monetary Theory of Production: Tradition and Perspectives, Palgrave: Macmillan, Cambrdige, 2005.
[18] I. Fisher, The Purchasing Power of Money: Its Determination and Relation to Credit, Interest, and Crises, The Macmillan Company, New York, 1911.
[19] R.F. Harrod, Money, Macmillan, London, 1969.