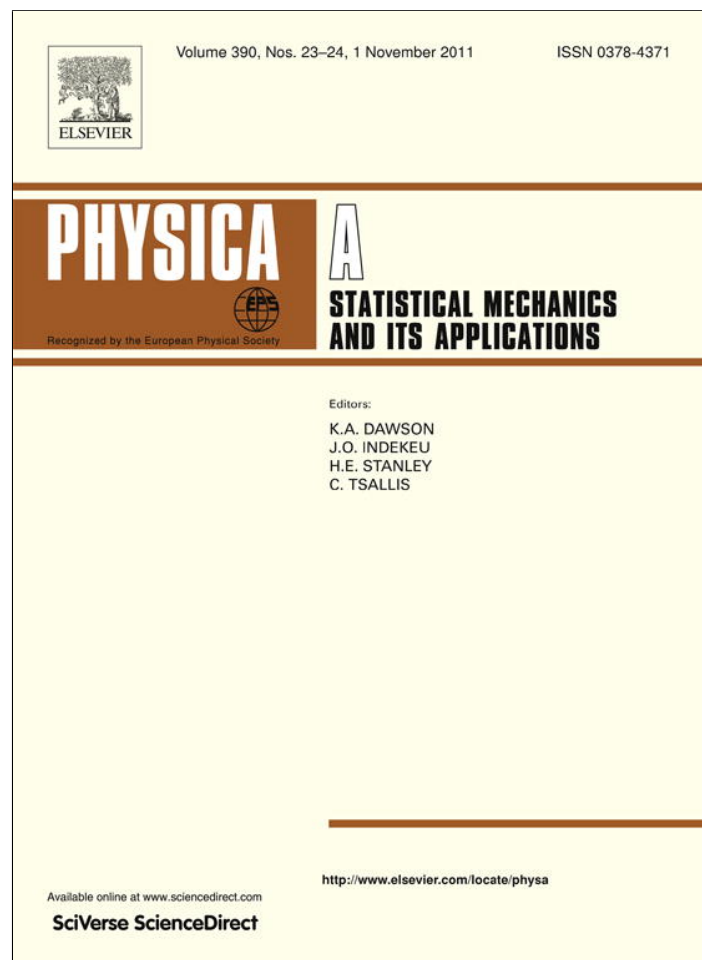


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Pulsation of the growth rate of output and technology

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ABSTRACT

The pulsation of the growth rate of output is considered from the point of view of technological theory of production, which is based on the law of substituting for human, muscle- and brain-based work. It is shown that the pulsation is connected with pulsation of technological characteristics of installed production equipment. The case of the US economy is considered.

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1. Introduction

Economic growth is usually accompanied by pulsation of production activity, in particular, by pulsation of the growth rate of output. In line with 'stylised' facts of the smooth economic growth, the understanding of the mechanics of cycle phenomena is important to test the existing theories of economic growth. The short-run variations of economic activity, which are termed as business cycles, are observed in every economy [1]. At the world level, research of empirical data for the GDP dynamics, employing spectral analysis, has confirmed the presence of famous cycles of a period of 50–60 years, so-called Kondratiev waves, at an acceptable level of statistical significance [2]. The authors also have detected shorter cycles, giving additional motivation for studying all sorts of cycles.

The conventional base for investigation of the economic growth is the two-factor Solow's theory [3], suffering from many deficiencies, when applied to a separate economy. The theory predicts that the growth rate of GDP should converge towards equality, with wealthy countries experiencing smaller relative growth rates than poor countries; while considering the cross-country comparison for long time horizon of 50 years, the opposite has been found for economic-wealth data [4]. Recently, Shao et al. [5] investigated the "convergence" of countries' per capita gross domestic products, assuming that the past twenty-year trends continue to hold in the future. They found that after approximately 30 years, both developing and developed EU countries will have comparable values of their per capita income, while the picture is different at the world level: between 1960 and 2009, the cross-country income differences increased over time. However, the economies should not be considered isolated from the rest of the world. Due to globalisation and capital transfer, one may expect that the interaction, one of the indicators of which can be the amount of national debt [6], affects the growth rate of GDP. Petersen et al. [6] found that, in time evolution of national public debt there is convergence—initially less-indebted countries increase their debt more quickly than initially more-indebted countries. One may expect also that the interaction affects different cycles and make them shorter or longer than they used to be without interaction. However, due to lack of data this hypothesis will be checked in years to come.

This investigation is restricted with studying the short-run variations of economic activity of a separate economy (the business cycles). It is known that consistent pattern of cycle phenomena can be understood in the framework of Solow

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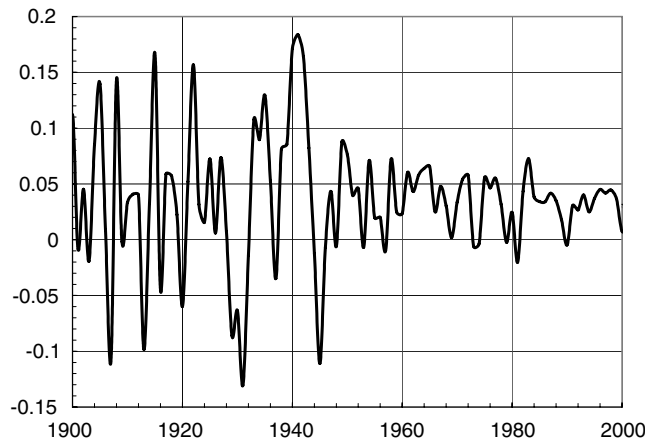


Fig. 1. The rate of growth of the US GDP. The rate of growth of Gross Domestic Product for the US economy shows a pulsating character of progress of production. This and the following graphs are based on the annual statistics collected earlier [10].

neoclassical theory of growth [7,8]. However, the Solow’s theory itself [3] is an object of severe criticism, and now one has different versions of corrected growth theories [9]. In this paper, we are describing the cycle phenomena from a point of view of the technological theory of production, the main relations of which are discussed in Appendix. It was demonstrated earlier [10,11], that the theory describes ‘stylised’ facts of the smooth economic growth without any forced arguments. We shall show on an example of well documentary dynamics of the US economy, that pulsation in economic growth can be reduced to change of technology, exploited by production system.

2. Empirical evidence

The investigators observe, that the growth of output (Gross Domestic Product) shows some deviations from some slowly varying path or trend, considering these deviations as fluctuations or business cycles, and have propose the methods of detrending [8]. Apparently, business cycle facts are sensitive to the procedure of detrending, but, if, suppose, the trend curve is smooth, the growth rate of output could be a good exposition of business cycles, which are shown in Fig. 1 for the US economy. One can see that the period of short pulsation of the growth rate of the Gross Domestic Product of the USA economy makes about four years.

Simultaneously with the growth rate of GDP, other variables, such as employment, investment, consumption, are changing also [7,8,12]. Further, a special attention to technological characteristics of economic systems will be paid.

The approach that is described in Appendix allows one introducing the quality of investment that is technological characteristics of production equipment, as they are defined by Eqs. (A.1). The coefficients of labour and energy requirement can be manipulated both in the dimensional, and dimensionless (with a bar on the top) forms

$$\bar{\lambda} = \frac{K}{L}\lambda, \quad \bar{\varepsilon} = \frac{K}{P}\varepsilon.$$

Production equipment to be installed is characterised with the technological coefficients $\bar{\lambda}$ and $\bar{\varepsilon}$. The dimensional coefficients λ and ε are characteristics of introduced technology and determine the required amount of labour and substitutive work per unit of investment. If the non-dimensional quantities are less than unity, it means that labour-saving and capital-service-saving technologies are being introduced at the time. These quantities for the US economy are depicted on Fig. 2.

3. Mechanism of business cycle

Prescott [8] presented the results of analysis of cycle phenomena based on the standard Solow growth theory that, as it is known, has been introducing technical progress artificially to describe ‘stylised’ facts of economic growth. The technological theory of economic growth includes the description of technology as a part of aggregative growth model (see Appendix) and will be exploited to establish the connection of pulsation of output with other quantities.

According to formula (A.14), one can write, neglecting the changes of productivity of capital ξ , for increment of the growth rate of output,

$$\Delta \left(\frac{1}{Y} \frac{dY}{dt} \right) = \frac{1}{\bar{\lambda}} \Delta \nu - \frac{\nu + \mu}{\bar{\lambda}^2} \Delta \bar{\lambda} + \frac{1 - \bar{\lambda}}{\bar{\lambda}} \Delta \mu. \tag{1}$$

Decrease in coefficient of labour requirement give increase in the growth rate of output, if the demand of labour in production is constant, $\Delta \nu = 0$. In the general case, the second equation from the set (A.1) gives for the increment of labour

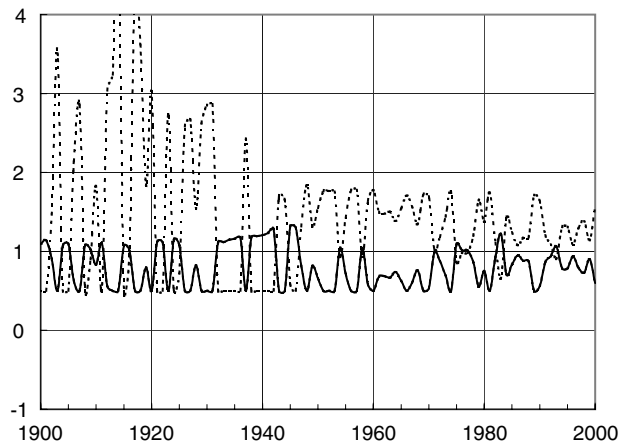


Fig. 2. Technological coefficients for the US economy. The labour (solid lines) and substitutive work (dotted lines) requirements are estimated due to Eqs. (A.1) at assumption that there are limiting bottom values of the technological coefficients: $\bar{\lambda}_0 = \bar{\varepsilon}_0 = 0.5$. The method of estimation is described in Refs. [13,14].

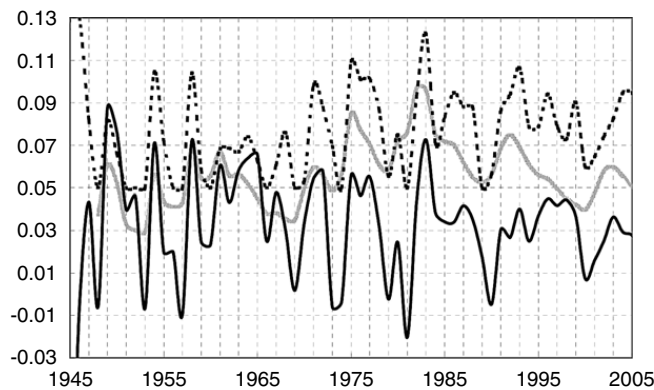


Fig. 3. Comparison of pulsation of different quantities. The picture shows that pulsation of the growth rate of Gross Domestic Product for the US economy (solid line) coincides with pulsation of the coefficient of labour requirement (the dashed line, values of the coefficient are divided by 10) and index of unemployment (the dotted line).

demand

$$\Delta v = \Delta \left(\frac{1}{L} \frac{dL}{dt} \right) = \frac{I}{K} \Delta \bar{\lambda} + \bar{\lambda} \Delta \left(\frac{I}{K} \right) - \Delta \mu. \quad (2)$$

The above equations shows, that increment of output can be reduced to increments of two quantities: the ratio of investment to the existing capital I/K and coefficient of labour requirement

$$\Delta \left(\frac{1}{Y} \frac{dY}{dt} \right) = \Delta \left(\frac{I}{K} \right) + \frac{1}{\bar{\lambda}} \left(\frac{I}{K} - \frac{\nu + \mu}{\bar{\lambda}} \right) \Delta \bar{\lambda} - \Delta \mu \approx \Delta \left(\frac{I}{K} \right) + \frac{1}{\bar{\lambda}} \frac{I}{K} \Delta \bar{\lambda}. \quad (3)$$

Fig. 3 shows the synchrony of pulsation of output and labour requirement, but one should consider the case more thoroughly, comparing increments of output, on one side, and increments of the ratio investment to capital and the coefficient of labour requirement, on the other side. These quantities are depicted on Fig. 4. One can find that within interval of years 1945–2005, the mean square deviation of the ratio I/K , which is 0.0008, does not correspond to the mean square deviation of the output, which is 0.076. Besides, the increment of the ratio anticorrelate with the increment of output (coefficient of correlation -0.40). The coefficient of labour requirement and output correlate with coefficient of correlation 0.66; the mean square deviation of labour requirement is equal to 0.36. So, one can find that the first term is approximately 40 times less than the second term, taking into account the multipliers in Eq. (3). Neglecting the first term in Eq. (3) in comparison with the second term, one has a simple relation

$$\Delta \left(\frac{1}{Y} \frac{dY}{dt} \right) \approx \frac{1}{\bar{\lambda}} \frac{I}{K} \Delta \bar{\lambda}. \quad (4)$$

This means, that changes of output follow the changes of labour requirement, as it is illustrated on Fig. 4.

The condition, that labour requirement $\bar{\lambda}$ is less than unity, shows, that efforts of workers partially are replaced with work of the machines movable by outer energy sources; it is a typical situation for the US economy in the second half of

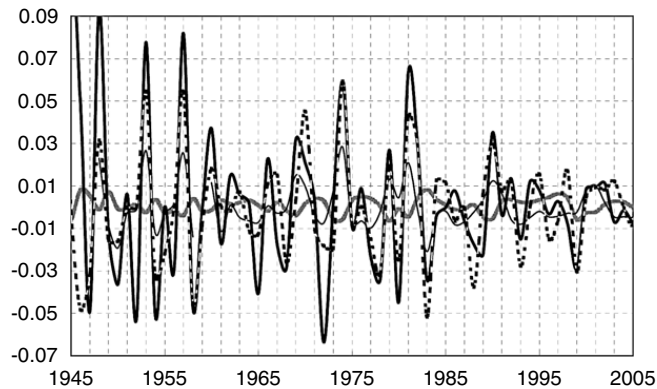


Fig. 4. Comparison of the increment of the growth rate of output with increments of different quantities. The picture shows the increment of the growth rate of Gross Domestic Product for the US economy (thick solid line), increment of unemployment (thin solid line), increment of labour requirement (the dashed line, values of the coefficient are divided by 10) and increment of the ratio of investment to capital (the dotted line). The pulsation of the coefficient of labour requirement correlate with the pulsation of output with coefficient of correlation 0.66, while pulsation of the ratio do not correlate with the pulsation of output (coefficient of correlation -0.40).

the twentieth century. The existence of pulsation of output can be connected with existence of the alternative types of functioning of production system, which were described in Appendix A.3. In the considered period for production of the USA economy, the second and the third cases (see Appendix A.3) are realised, that is, the processes are running at deficiency of labour and abundance of investments, substitutive work and raw materials, when $\frac{d\bar{\lambda}}{dt} < 0$; or at deficiency of substitutive work and abundance of investments, work and raw materials, when $\frac{d\bar{\lambda}}{dt} > 0$.

To describe an ideal cycle, one can start from the point, where the coefficient of labour requirement has its minimum value. In the case, when $\frac{d\bar{\lambda}}{dt} > 0$, the production system is experienced deficiency of substitutive work, whereas the offering of working places is restricted, and, at small values of the coefficient of labour requirement, the index of unemployment starts to increase. The coefficient of labour requirement is also growing, according to Eq. (A.8), the production system attracts more labour, but it appears insufficient to decrease in the index of unemployment immediately: the index grows simultaneously with the technological coefficient. The growth of unemployment stops, when production system succeeded in using all extra offering of labour, and the technological coefficient reaches its potential value at $\frac{d\bar{\lambda}}{dt} = 0$. The situation is balanced at the peak of unemployment, and, at this point, the change of the growth mode has occurred. Further, when $\frac{d\bar{\lambda}}{dt} < 0$, the production system is functioning at deficiency of labour, and the production system are able to use all available resources of workers: the index of unemployment decreases simultaneously with decrease in labour demand, until at some point in time a new balance occurs at $\frac{d\bar{\lambda}}{dt} = 0$, where the mode of functioning of the production system is changing again: a new cycle begins.

To design a mathematical model of ideal business cycle, apparently, apart of output, technological coefficients and labour demand, one has to include some other variables, first of all, labour supply and wage, and refer to some relations between output, wages and labour supply.

There remain many reasons for the observable real cycles not to be ideal (see Fig. 3). However, empirical data for the US economy confirm the general patterns of business cycle phenomena: the changes of the coefficient of labour requirement and the index of unemployment correlate with coefficient of correlation 0.75. The period of a cycle is connected with the mechanism of propagation of exploited technologies and for the US economy is equal to approximately four years.

4. Conclusion

The considered approximation of a national economy as a single sector allows describing dynamics of short cycles in the US economy in the last century from the point of view of production, confirming the description by Prescott [8]. The technological theory of production appears to be a proper organising structure for the growth and business cycle facts: it integrates growth and business cycle theories.

Apparently, it is possible also to explain the observable longer cycles [2] from the point of view of production and changes of modes of development. In this case, one can use a model of the national economy consisting of many sectors, whereas each sector is characterised by the technological coefficients with different times of propagations, so that it gives a wider set of modes of development. For the appropriate analysis it is necessary to consider the empirical situation in the production sectors and, except for time series for GDP, to involve into discussion time series for expenditures of labour, substitutive work, capital and investment for the considered sectors. The resulting picture appears to be rather complex.

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Appendix. The main relations of the technological theory of economic growth

Production factors are some universal characteristics of production processes [9] and can be considered as original sources of value. The choice of a set of production factors is decisive for the theory. In classical political economy (Adam Smith, Karl Marx, David Ricardo), the production factor is the human work (labour) L , which is used to be measured in working hours. Neoclassical practice [3,15] adds capital K , which is a money estimate of production equipment, that is value of the collection of all energy-conversion machines and information processing equipment, plus ancillary structures to contain and move them, including residential housing, when one considers capital in a wider sense. The further development, reviewed by Aghion and Howitt [9], showed the necessity of extra factors. Beaudreau [16,17], Pokrovskii [10,13,18] and Ayres with collaborators [19,20] explored the hypothesis, that energy delivered to animate production equipment, is one of the three major aggregate factors of production in modern industrialised economies.

Apparently, true work of production equipment, that is really matter in the problem, is a small part of the total amount of consumption of primary energy carriers E , which are the remains of the former biospheres: wood, coal, oil, direct and indirect solar energy (in form of wind, water, tides), energy of fission and fusion of nuclei. Properly refined true substitutive work P , by definition, is work in thermodynamic sense, designed to animate production equipment [10,16–18]; it is much less than the total consumption of energy inputs. Three production factors: labour L , capital K and substitutive work P are considered as independent variables, while substitutive work P and labour L inputs are substitutes to each other and capital K has to be considered to be a complement to work (L and P) of the production equipment. A systematic description of the theory is presented in the monograph [14].

A.1. Dynamics of production factors

The exploited technology determines that one needs a certain amount of efforts of human being L and a certain amount of capital services P to produce something. The expansion of production, characterised by changes of the accumulated value, requires additional labour and substitutive work, so that dynamics of the production factors can be written [10,13] as the balance equations

$$\frac{dK}{dt} = I - \mu K, \quad \frac{dL}{dt} = \left(\bar{\lambda} \frac{I}{K} - \mu \right) L, \quad \frac{dP}{dt} = \left(\bar{\varepsilon} \frac{I}{K} - \mu \right) P. \quad (\text{A.1})$$

The first terms on the right side of these relations describe the increase in the quantities caused by gross investment I , which is a part of the output accumulated in a material form of production equipment. The second terms on the right side of Eqs. (A.1) reflect the decrease in the corresponding quantities due to the removal of a part of the production equipment from the service with the depreciation coefficient μ .

Let us note that the simple relations (A.1) were written on the simplifying assumption that the characteristics of the equipment do not change after its installation, which implies that the all coefficients of depreciation in Eqs. (A.1) are equal, otherwise the dynamic equations takes a more complicated form. It is not difficult to write a more general relation [14].

A.2. The three-factor production function

According to classical and neo-classical tradition, production of value Y can be reduced to the production factors, which are considered as sources of value. A function of production factors, have to satisfy some requirements, which help to determine its form.¹ Taking into account the discussed properties of production factors, the production function for our case can be specified [10,11] as:

$$Y = \begin{cases} \xi K, & \xi > 0 \\ Y_0 \frac{L}{L_0} \left(\frac{L_0}{L}, \frac{P}{P_0} \right)^\alpha, & 0 < \alpha < 1 \end{cases} \quad (\text{A.2})$$

where Y_0 , L_0 and P_0 correspond to output, labour and substitutive work in the base year. The productivity of the capital stock ξ and the index α in Eq. (A.2) are parameters of the production system itself, and their derivatives are related to each other and to the characteristics of production system [14, Chapter 6, Equation 6.17]

$$\frac{1}{\xi} \frac{d\xi}{dt} = \ln \left(\frac{L_0}{L} \frac{P}{P_0} \right) \frac{d\alpha}{dt}, \quad \alpha = \frac{1 - \bar{\lambda}}{\bar{\varepsilon} - \bar{\lambda}}. \quad (\text{A.3})$$

This formulation corresponds to the two complementary descriptions of the production of value. The first one relates output to the amount of production equipment (capital stock) while the second describes the process of production through property of the same equipment to attract labour and energy (labour and capital services). The first line in (A.2) is analogous

¹ The discussion of effect of universality and homogeneity on the form of production function can be found in Section 6.3 of the monograph [14].

to the production technology found in the Harrod–Domar growth model [21–24], while the function in the second line resembles the Cobb–Douglas [15] production function.²

In the multi-sector approach (the input–output model), changes in the technological index are related to aggregate sectoral technological change and the difference in the growth rates across sectors, as can be seen from Equation 8.21 in Chapter 8 of the monograph [14]. The technological index itself can be estimated using all available information about the technological performance of the production system. Moreover, a condition regarding the optimal use of production factors enables us to establish a relation between the parameter α on one hand and the shared costs of production factors on the other [10,13]. This provides the different means of estimating of the technological index.

A.3. Three modes of development

The real investments are determined by the assumption that the production system is driven by available production factors. In any case, the rates of real growth do not exceed the rates of potential growth, given as functions of time

$$\tilde{\delta}(t) \geq \delta = \frac{1}{K} \frac{dK}{dt}, \quad \tilde{\nu}(t) \geq \nu = \frac{1}{L} \frac{dL}{dt}, \quad \tilde{\eta}(t) \geq \eta = \frac{1}{P} \frac{dP}{dt}. \quad (\text{A.4})$$

This, taking into account Eqs. (A.1), determines the restrictions for investments in the production sector

$$I \leq (\mu + \tilde{\delta})K, \quad I \leq \frac{\mu + \tilde{\nu}}{\lambda}L, \quad I \leq \frac{\mu + \tilde{\eta}}{\varepsilon}P. \quad (\text{A.5})$$

The real investments are determined by a competition between potential investments on one side and labour and energy supplies on the other side. One can assume that the production system tries to swallow up all available production factors. In this case, one ought to write for investments

$$I = (\delta + \mu)K = \min \begin{cases} (\tilde{\delta} + \mu)K \\ (\tilde{\nu} + \mu)K/\bar{\lambda} \\ (\tilde{\eta} + \mu)K/\bar{\varepsilon} \end{cases}. \quad (\text{A.6})$$

According to three lines of this relation, one can define three modes of economic development for which we have different formulae for calculation. As it is seen from Eqs. (A.1) and (A.6), the rates of real growth can be calculated for three modes as

$$\begin{aligned} \delta = \tilde{\delta}, \quad \nu = (\tilde{\delta} + \mu)\bar{\lambda} - \mu, \quad \eta = (\tilde{\delta} + \mu)\bar{\varepsilon} - \mu, \\ \delta = (\tilde{\nu} + \mu)\frac{1}{\bar{\lambda}} - \mu, \quad \nu = \tilde{\nu}, \quad \eta = (\tilde{\nu} + \mu)\frac{\bar{\varepsilon}}{\bar{\lambda}} - \mu, \\ \delta = (\tilde{\eta} + \mu)\frac{1}{\bar{\varepsilon}} - \mu, \quad \nu = (\tilde{\eta} + \mu)\frac{\bar{\lambda}}{\bar{\varepsilon}} - \mu, \quad \eta = \tilde{\eta}. \end{aligned} \quad (\text{A.7})$$

The first set of equations is valid in the case of lack of investment, abundance of labour, energy and raw materials. The second line is valid in the case of lack of labour, abundance of investment, energy and raw materials. The last line of equations is valid in the case of lack of energy, abundance of investment, labour and raw materials.

A.4. Dynamics of technological coefficients

To find out the law of the time dependence of the technological coefficients, one can refer to the restrictions on investments. One can assume that there are internal changes, which lead to changes of technological coefficients, whereas the production system tries to use all available resources. These processes are connected with the propagation of known technologies. Assuming, that the technological coefficients have tendencies to change in such a way that the inequalities in conditions (A.5) are trending to turn into the equalities, in the first approximation, one can get [13] the equations for the technological coefficients

$$\frac{d\bar{\lambda}}{dt} = -\frac{1}{\tau} \left(\bar{\lambda} - \frac{\tilde{\nu} + \mu}{\tilde{\delta} + \mu} \right), \quad (\text{A.8})$$

$$\frac{d\bar{\varepsilon}}{dt} = -\frac{1}{\tau} \left(\bar{\varepsilon} - \frac{\tilde{\eta} + \mu}{\tilde{\delta} + \mu} \right), \quad (\text{A.9})$$

² It bears reminding that in the conventional neoclassical approach, capital plays two distinctive roles: capital stock as the value of production equipment and capital service as a substitute for labour. These roles are ascribed to different variables in our theory: Eq. (A.2) contains substitutive work P as a capital service and capital stock K as a measure of amount of production equipment.

where τ is time of crossover from one technological situation to another, when external parameters \tilde{v} and $\tilde{\eta}$ change. The process is determined by internal processes of attracting of the proper technology.

Note, that now one can formulate attributes of each of three modes in terms of time dependence of the technological coefficients

$$\frac{d\bar{\lambda}}{dt} > 0 \quad \text{and} \quad \frac{d\bar{\varepsilon}}{dt} > 0, \tag{A.10}$$

$$\frac{d\bar{\lambda}}{dt} < 0 \quad \text{and} \quad \frac{d\bar{\varepsilon}}{dt} > 0, \tag{A.11}$$

$$\frac{d\bar{\lambda}}{dt} > 0 \quad \text{and} \quad \frac{d\bar{\varepsilon}}{dt} < 0. \tag{A.12}$$

The first line of these relations is valid in the case of lack of investment, abundance of labour, energy and raw materials. The second line is valid in the case of lack of labour, abundance of investment, energy and raw materials. The last line of equations is valid in the case of lack of energy, abundance of investment, labour and raw materials.

A.5. The growth rate of output

After having differentiated equations (A.2), the growth rate of output can be written in the form of two alternative expressions

$$\frac{1}{Y} \frac{dY}{dt} = \begin{cases} \frac{1}{K} \frac{dK}{dt} + \frac{1}{\xi} \frac{d\xi}{dt}, \\ (1 - \alpha) \frac{1}{L} \frac{dL}{dt} + \alpha \frac{1}{P} \frac{dP}{dt} + \ln \left(\frac{L_0 P}{L P_0} \right) \frac{d\alpha}{dt}. \end{cases} \tag{A.13}$$

The first terms in the first and second lines present contribution to growth due to the growth of production factors: capital, labour and substitutive work, the last ones—due to the change of the production system itself; the change of quantities ξ and α is connected with the technological and structural changes. The last terms cannot be reduced to any function of production factors.

Exploiting now Eqs. (A.7) for the growth rates of production factors in three possible cases, one see that Eqs. (A.13) reduce to a universal expression for the growth rate of output

$$\frac{1}{Y} \frac{dY}{dt} = \frac{\nu + (1 - \bar{\lambda})\mu}{\bar{\lambda}} + \frac{1}{\xi} \frac{d\xi}{dt}. \tag{A.14}$$

The labour requirement $\bar{\lambda}$ appears to be the most important quantity, which determines the change of productivity of labour. If $\bar{\lambda} = 1$, variations in technology do not occur, labour productivity is constant, and the increment of output is connected only with an increase in human's efforts. The human efforts are, certainly, the main motive power, but, under condition of $\bar{\lambda} < 1$ the workers' efforts partially are replaced with work of the machines movable by outer energy sources, and the labour productivity increases. This is a general description of influence of scientific and technological progress, which is naturally entered in a picture of progress of mankind.

Increase in labour productivity cannot be understood without taking into account the phenomenon accompanying progress of production, — attraction of natural energy sources (animals, wind, water, coal, oil and others) for performance of works that replace efforts of the humans in production. The developing of machine technologies appears to give increase, via effect of substitution, in labour productivity. A progressively greater amount of energy is utilised by human societies via improvements in technology.

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